Quick Review 6.3

1. **(a)** $\overrightarrow{OA} = \langle -3, -2 \rangle$

(b) $\overrightarrow{OB} = \langle 4, 6 \rangle$

(c) $\overrightarrow{AB} = \langle 4 - (-3), 6 - (-2) \rangle = \langle 7, 8 \rangle$

2. **(a)** $\overrightarrow{OA} = \langle -1, 3 \rangle$

(b) $\overrightarrow{OB} = \langle 4, -3 \rangle$

(c) $\overrightarrow{AB} = \langle 4 - (-1), -3 - 3 \rangle = \langle 5, -6 \rangle$

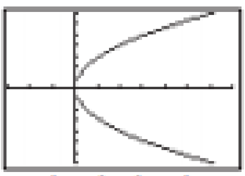
3. $m = \dfrac{6 - (-2)}{4 - (-3)} = \dfrac{8}{7}$

$y + 2 = \frac{8}{7}(x + 3)$ or $y - 6 = \frac{8}{7}(x - 4)$

4. $m = \dfrac{-3 - 3}{4 - (-1)} = -\dfrac{6}{5}$

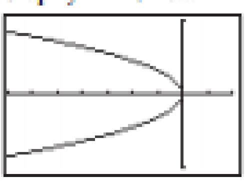
$y - 3 = -\frac{6}{5}(x + 1)$ or $y + 3 = -\frac{6}{5}(x - 4)$

5. Graph $y = \pm\sqrt{8x}$.



[−3, 7] by [−7, 7]

6. Graph $y = \pm\sqrt{-5x}$.



[−7, 2] by [−7, 7]

7. $x^2 + y^2 = 4$

8. $(x + 2)^2 + (y - 5)^2 = 9$

9. $\dfrac{600 \text{ rotations}}{1 \text{ min}} \cdot \dfrac{1 \text{ min}}{60 \text{ sec}} \cdot \dfrac{2\pi \text{ rad}}{1 \text{ rotation}} = 20\pi \text{ rad/sec}$

10. $\dfrac{700 \text{ rotations}}{1 \text{ min}} \cdot \dfrac{1 \text{ min}}{60 \text{ sec}} \cdot \dfrac{2\pi \text{ rad}}{1 \text{ rotation}} = \dfrac{70}{3}\pi \text{ rad/sec}$

Section 6.3 Exercises

1. **(b)** [−5, 5] by [−5, 5]

2. **(d)** [−5, 5] by [−5, 5]

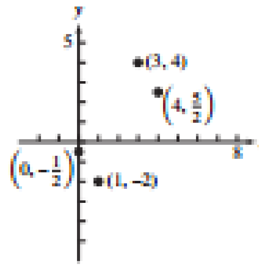
3. **(a)** [−5, 5] by [−5, 5]

4. **(c)** [−10, 10] by [−12, 10]

5. **(a)**

t	−2	−1	0	1	2
x	0	1	2	3	4
y	$-\frac{1}{2}$	−2	undef.	4	$\frac{5}{2}$

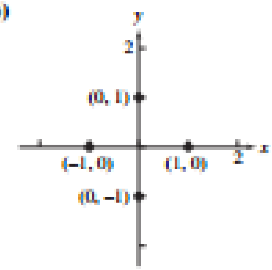
(b)



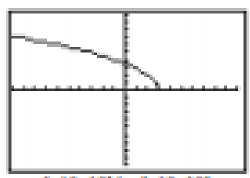
6. **(a)**

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	1	0	−1	0	1
y	0	1	0	−1	0

(b)

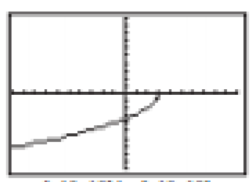


7.



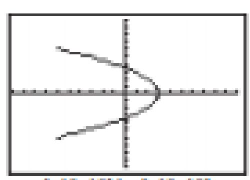
[−10, 10] by [−10, 10]

8.



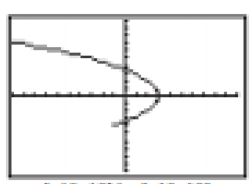
[−10, 10] by [−10, 10]

9.



[−10, 10] by [−10, 10]

10.



[−10, 10] by [−10, 10]

11. $x = 1 + y$, so $y = x - 1$: line through $(0, -1)$ and $(1, 0)$

12. $t = y - 5$, so $x = 2 - 3(y - 5)$; $y = -\frac{1}{3}x + \frac{17}{3}$: line through $\left(0, \frac{17}{3}\right)$ and $(17, 0)$

13. $t = \frac{1}{2}x + \frac{3}{2}$, so $y = 9 - 4\left(\frac{1}{2}x + \frac{3}{2}\right)$; $y = -2x + 3$, $3 \le x \le 7$: line segment with endpoints $(3, -3)$ and $(7, -11)$

14. $t = y - 2$, so $x = 5 - 3(y - 2)$; $y = -\frac{1}{3}x + \frac{11}{3}$, $-4 \le x \le 8$: line segment with endpoints $(8, 1)$ and $(-4, 5)$

15. $x = (y - 1)^2$: parabola that opens to right with vertex at $(0, 1)$

16. $y = x^2 - 3$: parabola that opens upward with vertex at $(0, -3)$

17. $y = x^3 - 2x + 3$: cubic polynomial

18. $x = 2y^2 - 1$; parabola that opens to right with vertex at $(0, -1)$

19. $x = 4 - y^2$; parabola that opens to left with vertex at $(4, 0)$

20. $t = 2x$, so $y = 16x^3 - 3$: cubic, $-1 \le x \le 1$

21. $t = x + 3$, so $y = \frac{2}{x + 3}$, on domain: $-8 \le x \le 2$, $x \ne -3$

22. $t = x - 2$, so $y = \frac{4}{x - 2}$, $x \ge 4$

23. $x^2 + y^2 = 25$, circle of radius 5 centered at $(0, 0)$

24. $x^2 + y^2 = 16$, circle of radius 4 centered at $(0, 0)$

25. $x^2 + y^2 = 4$, three-fourths of a circle of radius 2 centered at $(0, 0)$ (not in Quadrant II)

26. $x^2 + y^2 = 9$, semicircle of radius 3, $y \ge 0$ only

27. $\overrightarrow{OA} = \langle -2, 5\rangle$, $\overrightarrow{OB}\langle 4, 2\rangle$, $\overrightarrow{OP} = \langle x, y\rangle$

$\overrightarrow{OP} - \overrightarrow{OA} = t(\overrightarrow{OB} - \overrightarrow{OA})$

$\langle x + 2, y - 5\rangle = t\langle 6, -3\rangle$

$x + 2 = 6t \Rightarrow x = 6t - 2$

$y - 5 = 3t \Rightarrow y = -3t + 5$

28. $\overrightarrow{OA}\langle -3, -3\rangle$, $\overrightarrow{OB} = \langle 5, 1\rangle$, $\overrightarrow{OP} = \langle x, y\rangle$

$\overrightarrow{OP} - \overrightarrow{OA} = t(\overrightarrow{OB} - \overrightarrow{OA})$

$\langle x + 3, y + 3\rangle = t\langle 8, 4\rangle$

$x + 3 = 8t \Rightarrow x = 8t - 3$

$y + 3 = 4t \Rightarrow y = 4t - 3$

For #29–32, many answers are possible; one or two of the simplest are given.

29. Two possibilities are $x = t + 3$, $y = 4 - \frac{7}{3}t$, $0 \le t \le 3$, or $x = 3t + 3$, $y = 4 - 7t$, $0 \le t \le 1$.

30. Two possibilities are $x = 5 - t$, $y = 2 - \frac{6}{7}t$, $0 \le t \le 7$, or $x = 5 - 7t$, $y = 2 - 6t$, $0 \le t \le 1$.

31. One possibility is $x = 5 + 3\cos t$, $y = 2 + 3\sin t$, $0 \le t \le 2\pi$.

32. One possibility is $x = -2 + 2\cos t$, $y = -4 + 2\sin t$, $0 \le t \le 2\pi$.

33. In Quadrant I, we need $x > 0$ and $y > 0$, so $2 - |t| > 0$ and $t - 0.5 > 0$. Then $-2 < t < 2$ and $t > 0.5$, so $0.5 < t < 2$. This is not changed by the additional requirement that $-3 \le t \le 3$.

34. In Quadrant II, we need $x < 0$ and $y > 0$, so $2 - |t| < 0$ and $t - 0.5 > 0$. Then ($t < -2$ or $t > 2$) and $t > 0.5$, so $t > 2$. With the additional requirement that $-3 \le t \le 3$, this becomes $2 < t \le 3$.

35. In Quadrant III, we need $x < 0$ and $y < 0$, so $2 - |t| < 0$ and $t - 0.5 < 0$. Then ($t < -2$ or $t > 2$) and $t < 0.5$, so $t < -2$. With the additional requirement that $-3 \le t \le 3$, this becomes $-3 \le t < -2$.

36. In Quadrant IV, we need $x > 0$ and $y < 0$, so $2 - |t| > 0$ and $t - 0.5 < 0$. Then $-2 < t < 2$ and $t < 0.5$, so $-2 < t < 0.5$. This is not changed by the additional requirement that $-3 \le t \le 3$.

37. (a) One good window is $[-20, 300]$ by $[-1, 10]$. If your grapher allows, use "Simultaneous" rather than "Sequential" plotting. Note that 100 yd is 300 ft. To show the whole race, use $0 \le t \le 13$ (upper limit may vary), since Ben finishes in $12.91\overline{6}$ sec. Note that it is the *process* of graphing (during which one observes Ben passing Jerry and crossing "the finish line" first), not the final product (which is two horizontal lines) which is needed; for that reason, no graph is shown here.

(b) After 3 seconds, Jerry is at $20(3) = 60$ ft and Ben is at $24(3) - 10 = 62$ ft. Ben is ahead by 2 ft.

38. (a) If your grapher allows, use "Simultaneous" rather than "Sequential" plotting. To see the whole race, use $0 \le t \le 5.1$ (upper limit may vary), since the faster runner reaches the flag after 5.1 sec. Note that it is the *process* of graphing, not the final product (which shows a horizontal line) which is needed; for that reason, no graph is shown here.

(b) The faster runner (who is coming from the left in the simulation) arrives at $t = 5.1$ sec. At this instant, the slower runner is 4.1 ft away from the flag; the slower runner doesn't reach the flag until $t = 5.\overline{5}$ sec. This can be observed from the simulation, or by solving algebraically $x_1 = 50$ and $x_2 = 50$.

39. (a) $y = -16t^2 + v_0t + s_0 = -16t^2 + 0t + 1000 = -16t^2 + 1000$

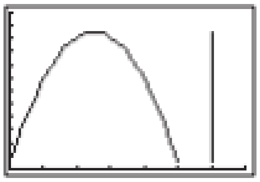
(b) Graph and trace: $x = 1$ and $y = -16t^2 + 1000$ with $0 \le t \le 6$, on the window $[0, 2]$ by $[0, 1200]$. Use something like 0.2 or less for Tstep. This graph will appear as a vertical line from $(1, 424)$ to $(1, 1000)$; it is not shown here because the simulation is accomplished by the *tracing*, not by the *picture*.

(c) When $t = 4$, $y = -16(4)^2 + 1000 = 744$ ft; the food containers are 744 ft above the ground after 4 sec.

40. (a) $y = -16t^2 + v_0t + s_0 = -16t^2 + 80t + 5$

(b) Graph and trace: $x = 6$ and $y = -16t^2 + 80t + 5$ with $0 \le t \le 5.1$ (upper limit may vary) on $[0, 7]$ by $[0, 120]$. This graph will appear as a vertical line from about $(6, 0)$ to about $(6, 105)$. Tracing shows how the ball begins at a height of 5 ft, rises to over 100 ft, then falls back to the ground.

(c) Graph $x = t$ and $y = -16t^2 + 80t + 5$ with $0 \le t \le 5.1$ (upper limit may vary).



[0, 7] by [0, 120]

(d) When $t = 4$, $y = -16(4)^2 + 80(4) + 5 = 69$ ft. The ball is 69 ft above the ground after 4 sec.

(e) From the graph in (b), when $t = 2.5$ sec, the ball is at its maximum height of 105 ft.

41. Possible answers:

(a) $0 < t < \frac{\pi}{2}$ (t in radians)

(b) $0 < t < \pi$

(c) $\frac{\pi}{2} < t < \frac{3\pi}{2}$

42. **(a)** Both pairs of equations can be changed to $x^2 + y^2 = 9$ — a circle centered at the origin with radius 3. Also, when one chooses a point on this circle and swaps the x- and y-coordinates, one obtains another point on the same circle.

(b) The first begins at the right side (when $t = 0$) and traces the circle counterclockwise. The second begins at the top (when $t = 0$) and traces the circle clockwise.

43. **(a)** $x = 400$ when $t \approx 2.80$ — about 2.80 sec.

(b) When $t \approx 2.80$ sec, $y \approx 7.18$ ft.

(c) Reaching up, the outfielder's glove should be at or near the height of the ball as it approaches the wall. If hit at an angle of 20°, the ball would strike the wall about 19.74 ft up (after 2.84 sec) — the outfielder could not catch this.

44. **(a)** No: $x = (120 \cos 30°)t$; this equals 350 when $t \approx 3.37$. At this time, the ball is at a height of $y = -16t^2 + (120 \sin 30°)t + 4 \approx 24.59$ ft.

(b) The ball hits the wall about 24.59 ft up when $t \approx 3.37$ (see **(a)**) — not catchable.

45. **(a)** Yes: $x = (5 + 120 \cos 30°)t$; this equals 350 when $t \approx 3.21$. At this time, the ball is at a height of $y = -16t^2 + (120 \sin 30°)t + 4 \approx 31.59$ ft.

(b) The ball clears the wall with about 1.59 ft to spare (when $t \approx 3.21$).

46. For Linda's ball, $x_1 = (45 \cos 44°)t$ and $y_1 = -16t^2 + (45 \sin 44°)t + 5$. For Chris's ball, $x_2 = 78 - (41 \cos 39°)t$ and $y_2 = -16t^2 + (41 \sin 39°)t + 5$. Find (graphically) the minimum of $d(t) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. It occurs when $t \approx 1.21$ sec; the minimum distance is about 6.60 ft.

47. No: $x = (30 \cos 70°)t$ and $y = -16t^2 + (30 \sin 70°)t + 3$. The dart lands when $y = 0$, which happens when $t \approx 1.86$ sec. At this point, the dart is about 19.11 ft from Tony, just over 10 in. short of the target.

48. Yes: $x = (25 \cos 55°)t$ and $y = -16t^2 + (25 \sin 55°)t + 4$. The dart lands when $y = 0$, which happens when $t \approx 1.45$ sec. At this point, the dart is about 20.82 ft from Sue, inside the target.

49. The parametric equations for this motion are $x = (v + 160 \cos 20°)\,t$ and $y = -16t^2 + (160 \sin 20°)t + 4$, where v is the velocity of the wind (in ft/sec) — it should be positive if the wind is in the direction of the hit, and negative if the wind is against the ball.
To solve this algebraically, eliminate the parameter t as follows:

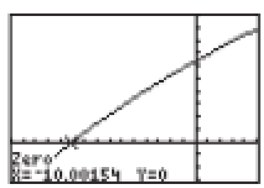
$$t = \frac{x}{v + 160 \cos 20°}. \text{ So } y = -16\left(\frac{x}{v + 160 \cos 20°}\right)^2 + 160 \sin 20°\left(\frac{x}{v + 160 \cos 20°}\right) + 4.$$

Substitute $x = 400$ and $y = 30$:

$$30 = -16\left(\frac{400}{v + 160 \cos 20°}\right)^2 + 160 \sin 20°\left(\frac{400}{v + 160 \cos 20°}\right) + 4.$$

Let $u = \frac{400}{v + 160 \cos 20°}$, so the equation becomes $-16u^2 + 54.72u - 26 = 0$. Using the quadratic formula, we find that $u = \frac{-54.72 \pm \sqrt{54.72^2 - 4(-16)(-26)}}{-32} \approx$ 0.57, 2.85. Solving $0.57 = \frac{400}{v + 160 \cos 20°}$ and $2.85 = \frac{400}{v + 160 \cos 20°}$, $v \approx 551.20$, $v \approx -10.00$. A wind speed of 551 ft/sec (375.7 mph) is unrealistic, so we eliminate that solution. So the wind will be blowing against the ball in order for the ball to hit within a few inches of the top of the wall.
To verify this graphically, graph the equation

$$30 = -16\left(\frac{400}{v + 160 \cos 20°}\right)^2 + 160 \sin 20°\left(\frac{400}{v + 160 \cos 20°}\right) + 4,$$ and find the zero.



[–15, 5] by [–3, 10]

50. Assuming the course is level, the ball hits the ground when $y = -16t^2 + (180 \sin \theta)t$ equals 0, which happens when $t = \frac{180 \sin \theta}{16} = 11.25 \sin \theta$ sec. At that time, the ball has traveled $x = (180 \cos \theta)t = 2025(\cos \theta)(\sin \theta)$ feet. The answers are therefore approximately:

(a) 506.25 ft.

(b) 650.82 ft.

(c) 775.62 ft.

(d) 876.85 ft.