6.2 Notes

Wednesday, January 11, 2017 4:38 PM

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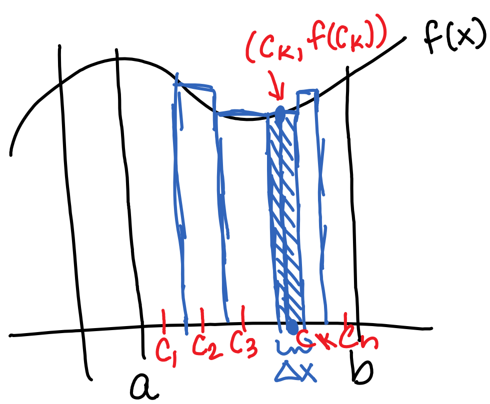
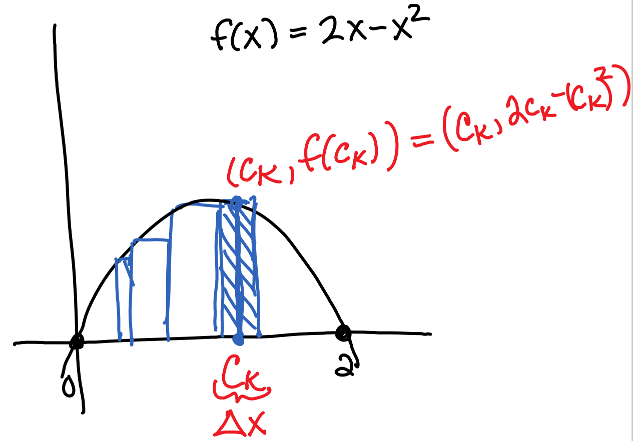
AP Calc AB 6.2: Definite Integrals & Area Under a Curve

Let's sketch 1 generic function and 1 specific function!



$[a,b]$



What does each of these represent?

1. $f(c_k) \bullet \Delta x$

ht • base = area of 1 Rectangle

1. $(2c_k - (c_k)^2) \bullet \Delta x$

ht • base = Area of 1 Rectangle

Sigma Notation

2. $\underbrace{\sum_{k=1}^{n}}_{\text{sum}} f(c_k) \bullet \Delta x$ ht · base

= total area of all the rectangles

2. $\underbrace{\sum_{k=1}^{n}}_{\text{Sum}} (2c_k - (c_k)^2) \bullet \Delta x$ ht · base

= total area of all the rectangles

$\boxed{\text{Riemann Sums}}$

3. $\lim_{\Delta x \to 0} \sum_{k=1}^{n} f(c_k) \bullet \Delta x$

as base Δx gets infinitely small, $n \to \infty$ (# of rectangles $\to \infty$) gives exact area

$= \int_a^b f(x)\,dx$

Sum a ht base

3. $\lim_{n \to \infty} \sum_{k=1}^{n} (2c_k - (c_k)^2) \bullet \Delta x$

as # rectangles $\to \infty$, Δx gets infinitely small – also gives exact area

$= \int_0^2 (2x - x^2)\,dx$

Sum 0 ht base

Definite Integral

$$\int_a^b f(x)\,dx$$

$f(x)$ = integrand

x = variable of integration

integral sign

$\int$ for Sum

a, b = lower, upper limits of integration

"The integral from a to b of $f(x)dx$."

Write in integral notation.

① $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \left(3(m_k)^2 - 2m_k + 5\right)\Delta x$ on interval $[-1, 3]$

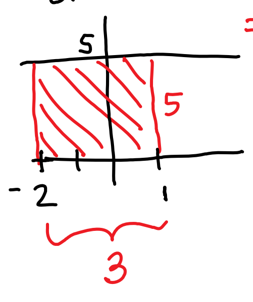
$$= \int_{-1}^{3} (3x^2 - 2x + 5)\,dx$$

② $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \sqrt{4 - (c_k)^2}\,\Delta x$ on interval $[0, 1]$

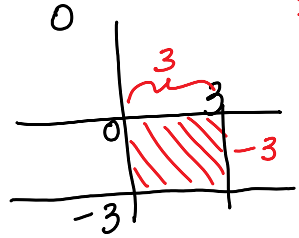
$$= \int_0^1 \sqrt{4 - x^2}\,dx$$

Evaluate the Integral. (Find the area under f(x) from a to b)

③ $\int_{-2}^{1} 5\,dx = (1-(-2))(5) = 3(5) = \boxed{15}$

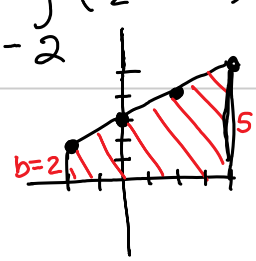


④ $\int_0^3 (-3)\,dt = 3(-3) = \boxed{-9}$



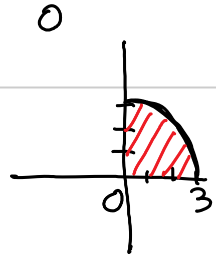
⑤ $\int_{-2}^{4} \left(\tfrac{1}{2}x + 3\right)dx$

$A = \frac{1}{2}(2+5)(6) = \boxed{21}$

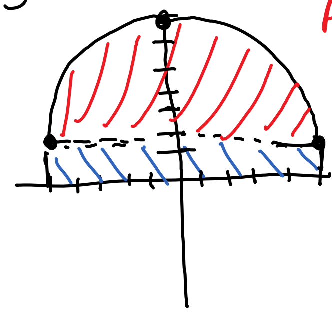


⑦ $\int_0^3 \sqrt{9 - x^2}\,dx$ ← half-circle w/radius = 3

$A = \frac{1}{4}\pi(3)^2 = \boxed{\frac{9\pi}{4}}$



(8) $\int_{-5}^{5} \left(2 + \sqrt{25 - x^2}\right) dx$



$A = \frac{1}{2}\pi(5)^2 + 2(10)$

$= \boxed{\frac{25\pi}{2} + 20}$