

4. If  $R = (-2, -1)$  and  $S = (2, 4)$ , then, using the HMT rule,

$$\overrightarrow{RS} = (2 - (-2), 4 - (-1)) = (4, 5).$$

- If  $P = (-3, -1)$  and  $Q = (1, 4)$ , then, using the HMT rule,

$$\overrightarrow{PQ} = (1 - (-3), 4 - (-1)) = (4, 5).$$

Both vectors represent  $(4, 5)$  by the HMT rule.

5.  $\overrightarrow{PQ} = (3 - (-2), 4 - 2) = (5, 2),$

$$|\overrightarrow{PQ}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

6.  $\overrightarrow{RS} = (2 - (-2), -8 - 5) = (4, -13),$

$$|\overrightarrow{RS}| = \sqrt{4^2 + (-13)^2} = \sqrt{185}$$

7.  $\overrightarrow{QR} = (-2 - 3, 5 - 4) = (-5, 1),$

$$|\overrightarrow{QR}| = \sqrt{(-5)^2 + 1^2} = \sqrt{26}$$

8.  $\overrightarrow{PS} = (2 - (-2), -8 - 2) = (4, -10),$

$$|\overrightarrow{PS}| = \sqrt{4^2 + (-10)^2} = \sqrt{116} = 2\sqrt{29}$$

9.  $2\overrightarrow{QS} = 2(2 - 3, -8 - 4) = (-2, -24),$

$$|2\overrightarrow{QS}| = \sqrt{(-2)^2 + (-24)^2} = \sqrt{580} = 2\sqrt{145}$$

10.  $(\sqrt{2})\overrightarrow{PR} = \sqrt{2}(-2 - (-2), 5 - 2) = (0, 3\sqrt{2}),$

$$|\sqrt{2}\overrightarrow{PR}| = \sqrt{0^2 + (3\sqrt{2})^2} = 3\sqrt{2}$$

11.  $3\overrightarrow{QR} + \overrightarrow{PS} = 3(-5, 1) + (4, -10) = (-11, -7),$

$$|3\overrightarrow{QR} + \overrightarrow{PS}| = \sqrt{(-11)^2 + (-7)^2} = \sqrt{170}$$

12.  $\overrightarrow{PS} - 3\overrightarrow{PQ} = (4, -10) - 3(5, 2) = (-11, -16),$

$$|\overrightarrow{PS} - 3\overrightarrow{PQ}| = \sqrt{(-11)^2 + (-16)^2} = \sqrt{377}$$

13.  $(-1, 3) + (2, 4) = (1, 7)$

14.  $(-1, 3) - (2, 4) = (-3, -1)$

15.  $(-1, 3) - (2, -5) = (-3, 8)$

16.  $3(2, 4) = (6, 12)$

17.  $2(-1, 3) + 3(2, -5) = (4, -9)$

18.  $2(-1, 3) - 4(2, 4) = (-10, -10)$

19.  $-2(-1, 3) - 3(2, 4) = (-4, -18)$

20.  $-(-1, 3) - (2, 4) = (-1, -7)$

21.  $\frac{\mathbf{u}}{|\mathbf{u}|} = \left\langle \frac{-2}{\sqrt{(-2)^2 + 4^2}}, \frac{4}{\sqrt{(-2)^2 + 4^2}} \right\rangle$

$$= -\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$$

22.  $\frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{1}{\sqrt{1^2 + (-1)^2}}, \frac{-1}{\sqrt{1^2 + (-1)^2}} \right\rangle$

$$= \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$$

23.  $\frac{\mathbf{w}}{|\mathbf{w}|} = \left\langle \frac{-1}{\sqrt{(-1)^2 + (-2)^2}}, \frac{-2}{\sqrt{(-1)^2 + (-2)^2}} \right\rangle$

$$= -\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j}$$

24.  $\frac{\mathbf{w}}{|\mathbf{w}|} = \left\langle \frac{5}{\sqrt{5^2 + 5^2}}, \frac{5}{\sqrt{5^2 + 5^2}} \right\rangle$

$$= \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$$

For #25–28, the unit vector in the direction of  $\mathbf{v} = (a, b)$  is

$$\frac{1}{|\mathbf{v}|} = \left\langle \frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}} \right\rangle$$

$$= \frac{a}{\sqrt{a^2 + b^2}}\mathbf{i} + \frac{b}{\sqrt{a^2 + b^2}}\mathbf{j}.$$

25. (a)  $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

(b)  $\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$

45. (a)  $\mathbf{v} = 10(\cos 70^\circ, \sin 70^\circ) \approx (3.42, 9.40)$

- (b) The horizontal component is the (constant) horizontal speed of the basketball as it travels toward the basket. The vertical component is the vertical velocity of the basketball, affected by both the initial speed and the downward pull of gravity.

26. (a)  $\left\langle -\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$

(b)  $-\frac{3}{\sqrt{13}}\mathbf{i} + \frac{2}{\sqrt{13}}\mathbf{j}$

27. (a)  $\left\langle -\frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}} \right\rangle$

(b)  $-\frac{4}{\sqrt{41}}\mathbf{i} + \left(-\frac{5}{\sqrt{41}}\right)\mathbf{j} = -\frac{4}{\sqrt{41}}\mathbf{i} - \frac{5}{\sqrt{41}}\mathbf{j}$

28. (a)  $\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

(b)  $\frac{3}{5}\mathbf{i} + \left(-\frac{4}{5}\right)\mathbf{j} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$

29.  $\mathbf{v} = (18 \cos 25^\circ, 18 \sin 25^\circ) \approx (16.31, 7.61)$

30.  $\mathbf{v} = (14 \cos 55^\circ, 14 \sin 55^\circ) \approx (8.03, 11.47)$

31.  $\mathbf{v} = (47 \cos 108^\circ, 47 \sin 108^\circ) \approx (-14.52, 44.70)$

32.  $\mathbf{v} = (33 \cos 136^\circ, 33 \sin 136^\circ) \approx (-23.74, 22.92)$

33.  $|\mathbf{u}| = \sqrt{3^2 + 4^2} = 5, \alpha = \cos^{-1}\left(\frac{3}{5}\right) \approx 53.13^\circ$

34.  $|\mathbf{u}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}, \alpha = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right) \approx 116.57^\circ$

35.  $|\mathbf{u}| = \sqrt{3^2 + (-4)^2} = 5, \alpha = 360^\circ - \cos^{-1}\left(\frac{3}{5}\right) \approx 306.87^\circ$

36.  $|\mathbf{u}| = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34},$

$$\alpha = 360^\circ - \cos^{-1}\left(\frac{-3}{\sqrt{34}}\right) \approx 239.04^\circ$$

37. Since  $(7 \cos 135^\circ)\mathbf{i} + (7 \sin 135^\circ)\mathbf{j} = (|\mathbf{u}| \cos \alpha)\mathbf{i} + (|\mathbf{u}| \sin \alpha)\mathbf{j}, |\mathbf{u}| = 7$  and  $\alpha = 135^\circ.$

38. Since  $(2 \cos 60^\circ)\mathbf{i} + (2 \sin 60^\circ)\mathbf{j} = (|\mathbf{u}| \cos \alpha)\mathbf{i} + (|\mathbf{u}| \sin \alpha)\mathbf{j}, |\mathbf{u}| = 2$  and  $\alpha = 60^\circ.$

For #39 and 40, first find the unit vector in the direction of  $\mathbf{u}$ . Then multiply by the magnitude of  $\mathbf{v}, |\mathbf{v}|.$

39.  $\mathbf{v} = |\mathbf{v}| \cdot \frac{\mathbf{u}}{|\mathbf{u}|} = 2 \left\langle \frac{3}{\sqrt{3^2 + (-3)^2}}, \frac{-3}{\sqrt{3^2 + (-3)^2}} \right\rangle$   
 $= 2 \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle = (\sqrt{2}, -\sqrt{2})$

40.  $\mathbf{v} = |\mathbf{v}| \cdot \frac{\mathbf{u}}{|\mathbf{u}|} = 5 \left\langle \frac{-5}{\sqrt{(-5)^2 + 7^2}}, \frac{7}{\sqrt{(-5)^2 + 7^2}} \right\rangle$

$$\approx 5(-0.58, 0.81) = (-2.91, 4.07)$$

41. A bearing of  $335^\circ$  corresponds to a direction angle of  $115^\circ.$   
 $\mathbf{v} = 530(\cos 115^\circ, \sin 115^\circ) \approx (-223.99, 480.34).$

42. A bearing of  $170^\circ$  corresponds to a direction angle of  $-80^\circ.$   
 $\mathbf{v} = 460(\cos(-80^\circ), \sin(-80^\circ)) \approx (79.88, -453.01).$

43. (a) A bearing of  $340^\circ$  corresponds to a direction angle of  $110^\circ.$   
 $\mathbf{v} = 325(\cos 110^\circ, \sin 110^\circ) \approx (-111.16, 305.40).$

(b) The wind bearing of  $320^\circ$  corresponds to a direction angle of  $130^\circ.$  The wind vector is  $\mathbf{w} = 40(\cos 130^\circ, \sin 130^\circ) \approx (-25.71, 30.64).$

Actual velocity vector:  $\mathbf{v} + \mathbf{w} \approx (-136.87, 336.04).$

Actual speed:  $\|\mathbf{v} + \mathbf{w}\| \approx \sqrt{136.87^2 + 336.04^2} \approx 362.84$  mph.