

The Law of Sines can be used to solve triangles when you know ASA, AAS.

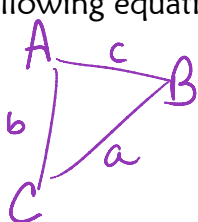
The Law of Cosines can be used to solve triangles when you know SAS, SSS.

(Either rule can be used for ASS, but remember that there could be 0, 1, or 2 triangles – we'll deal with that later.)

The Law of Cosines is called the “generalized Pythagorean Theorem.”

The Law of Cosines states:

In any  $\triangle ABC$  with angles A, B, and C opposite sides a, b, and c, respectively, the following equations are true:



$$a^2 = b^2 + c^2 - 2bccosA$$

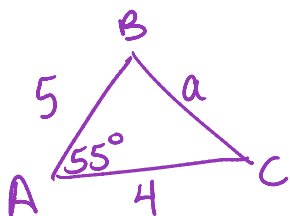
$$b^2 = a^2 + c^2 - 2accosB$$

$$c^2 = a^2 + b^2 - 2abcosC$$

**Examples:** Find the missing side.

1.  $\triangle ABC, b = 4, c = 5, m\angle A = 55^\circ$

SAS

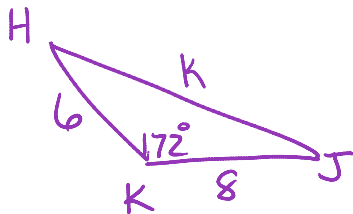


$$a^2 = 5^2 + 4^2 - 2(5)(4)\cos 55^\circ$$

$$a \approx 4.2$$

2.  $\triangle HJK, h = 8, j = 6, m\angle K = 172^\circ$

SAS



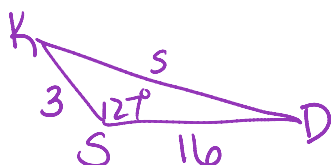
$$k^2 = 6^2 + 8^2 - 2(6)(8)\cos 172^\circ$$

$$k \approx 14.0$$

**Try it!** Find the missing side.

3.  $\triangle KSD, m\angle S = 127^\circ, k = 16, d = 3$

SAS

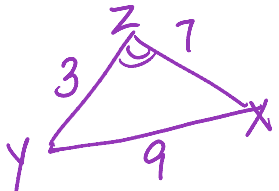


$$s^2 = 3^2 + 16^2 - 2(3)(16)\cos 127^\circ$$

$$s \approx 18.0$$

Find the angles of the triangle.

4.  $\Delta XYZ, x=3, y=7, z=9$   
SSS



To Find  $\angle Z$ :

$$9^2 = 3^2 + 7^2 - 2(3)(7)\cos Z$$

$$23 = -42\cos Z$$

$$-\frac{23}{42} = \cos Z$$

$$\angle Z = \cos^{-1}\left(-\frac{23}{42}\right) \approx 123.2^\circ$$

To Find  $\angle X$ :

$$3^2 = 7^2 + 9^2 - 2(7)(9)\cos X$$

$$-121 = -126\cos X$$

$$\frac{121}{126} = \cos X$$

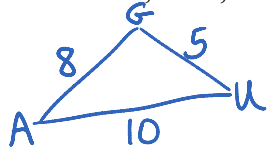
$$\angle X = \cos^{-1}\left(\frac{121}{126}\right) \approx 16.2^\circ$$

Subtract from  $180^\circ$  to find  $\angle Y$ :

$$\angle Y = 180 - \angle Z - \angle X \approx 40.6^\circ$$

Try it! Find the angles of the triangle.

5.  $\Delta AUG, a=5, u=8, g=10$



To Find  $\angle G$ :

$$10^2 = 8^2 + 5^2 - 2(8)(5)\cos G$$

$$-\frac{11}{80} = \cos G$$

$$\angle G = 97.9^\circ$$

To Find  $\angle U$ :

$$8^2 = 5^2 + 10^2 - 2(5)(10)\cos U$$

$$-\frac{61}{100} = \cos U$$

$$\angle U = 52.4^\circ$$

To Find  $\angle A$ :

$$\angle A = 180 - \angle G - \angle U$$

$$\angle A = 29.7^\circ$$

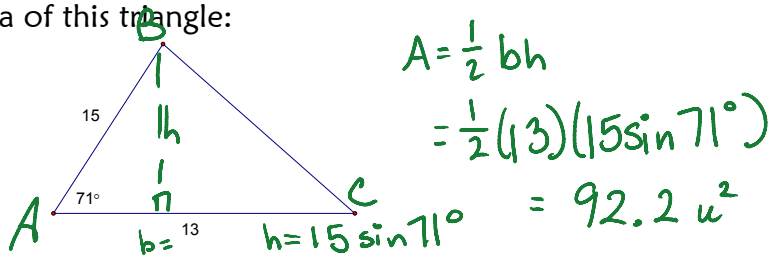
Assign p. 494

# 1, 3, 6, 9, 35, 38

### Area of a Triangle – 2 Formulas

Area of a Triangle =  $\frac{1}{2}bh$

Find the area of this triangle:



(any side can be base)

### Area of a Triangle

$$A_{\Delta} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

This formula is for when you know 2 sides and included angle SAS

If you know 3 sides of the  $\Delta$  (SSS) instead, you can use this formula from Geometry:

### Heron's Formula

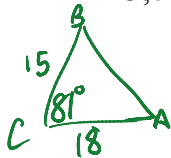
semi-perimeter  
 $s = \frac{a+b+c}{2}$

$$A_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)}$$

This formula is for when you know SSS

**Examples:** Find the area of the given triangle to the nearest 10<sup>th</sup>.

6.  $\Delta ABC, b=18, a=15, m\angle C = 81^\circ$

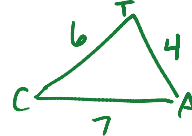


SAS

$$A_{\Delta} = \frac{1}{2}(18)(15)\sin 81^\circ$$

$$\approx \boxed{133.3 \text{ u}^2}$$

7.  $\Delta CAT, c=4, a=6, t=7$



SSS

$$s = \frac{6+4+7}{2} = 8.5$$

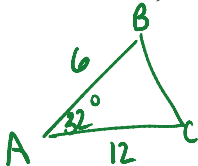
$$A_{\Delta} = \sqrt{8.5(8.5-6)(8.5-4)(8.5-7)}$$

$$\approx \boxed{12.0 \text{ u}^2}$$

Assign p. 494  
#17, 20, 21, 24

**Try it!** Find the area of the given triangle to the nearest 10<sup>th</sup>.

8.  $\Delta ABC, c=6, b=12, m\angle A = 32^\circ$

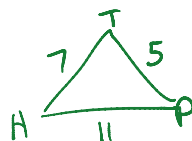


SAS

$$A_{\Delta} = \frac{1}{2}(6)(12)\sin 32^\circ$$

$$\approx \boxed{19.1 \text{ u}^2}$$

9.  $\Delta HPT, h=5, p=7, t=11$

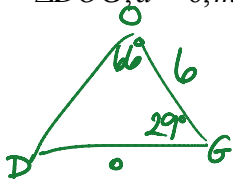


$$s = \frac{7+5+11}{2} = 11.5$$

$$A_{\Delta} = \sqrt{11.5(11.5-7)(11.5-5)(11.5-11)}$$

$$\approx \boxed{13.0 \text{ u}^2}$$

10.  $\Delta DOG, d=6, m\angle O = 66^\circ, m\angle G = 29^\circ$  (hint: how can you find the side you need first?)



ASA - Law of Sines

to find  $o$ .  
 $m\angle D = 85^\circ$

$$\frac{\sin 85^\circ}{6} = \frac{\sin 66^\circ}{o}$$

$$o = 5.5$$

$$A_{\Delta} = \frac{1}{2}(5.5)(6)\sin 29^\circ$$

$$\approx \boxed{8.0 \text{ u}^2}$$