5.6 Related Rates

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5.6a Notes Related Rates

Assume x and y are both functions of time. That is assume they are both variables with respect to time, or they both change over time.

Differentiate:

a) $y = 2x^2 + 5x$ with respect to x

$\frac{dy}{dx} = 4x + 5$

a) $y = 2x^2 + 5x$ with respect to t

$\frac{dy}{dt} = 4x \cdot \frac{dx}{dt} + 5 \cdot \frac{dx}{dt}$

Given $\boxed{y = xz}$

a) Find $\frac{dy}{dt}$ if z is a constant and x is a function of time. $\frac{dy}{dt} = z\frac{dx}{dt}$

b) Find $\frac{dy}{dt}$ if x is a constant and z is a function of time. $\frac{dy}{dt} = x\frac{dz}{dt}$

c) Find $\frac{dy}{dt}$ if x and z are both functions of time. $\frac{dy}{dt} = x\frac{dz}{dt} + z\frac{dx}{dt}$

product rule

Related rates are essentially about finding how as some quantities are changing with time, other quantities are changing with time.

Examples:

How the depth of water in a bathtub changes as the volume is changing.

How the volume of a balloon is changes as the radius is changing.

How the current in an electrical circuit changes as the voltage is changing.

Important Note: Pay attention to the sign of rates of change in related rates problems.

Example:

$\frac{dV}{dt}$ = the rate of change of the volume of water in a tub with respect to time.

a) If the water is flowing into the tub the rate of change is positive.
b) If the water is flowing out of the tub the rate of change is negative.

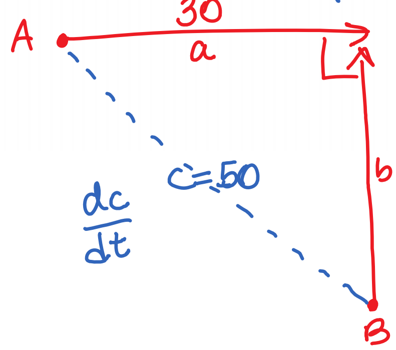
a) If the water is flowing into the tub the rate of change is ____________.

b) If the water is flowing out of the tub the rate of change is negative.

Examples

1. Two people are walking toward the same corner at 2ft/second and 4ft/second. At the time when the first person is 30ft from the corner and the second is 40ft from the corner, at what rate is the distance between them changing?

$\frac{da}{dt} = -2 \frac{ft}{sec}$



$\frac{db}{dt} = -4$ ft/sec

Pythagorean Thm

$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(30)(-2) + 2(40)(-4) = 2(50)\frac{dc}{dt}$

$-120 - 320 = 100\frac{dc}{dt}$

$-4.4 = \frac{dc}{dt}$

$\boxed{\frac{dc}{dt} = -4.4 \frac{ft}{sec}}$

2. A spherical balloon is losing volume at a rate of 20π cubic inches/second. How fast is the radius changing at the time the radius is 5 inches?

$\frac{dV}{dt} = -20\pi$ in³/sec

$r = 5$ in

Find: $\frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-20\cancel{\pi} = 4\cancel{\pi}(5)^2 \frac{dr}{dt}$

$-\frac{1}{5} = \frac{dr}{dt}$

$\frac{dr}{dt} = \boxed{-\frac{1}{5} \text{ in/sec}}$

Related Rates Solution Strategies

1. Understand the problem and identify the variables and constants along with the rates of change you are given and those you want to find.
2. Draw and label a picture. Assign variables and distinguish between variables and constants.
3. Write an equation relating the variables whose rate of change you are trying to find with variable(s) whose rate(s) of change you are given.
4. Differentiate both sides of the equation implicitly with respect to time.
5. Substitute values for any quantities that depend on time.
6. Solve for the wanted rate of change and interpret the solution.

3. A circular patch of forest is en fuego! When the radius is 80m, the fire spreads at a rate of 1000 square meters / minute. How quickly is the radius of the blaze growing at that moment?

$r = 80\text{ m}$

$\frac{dA}{dt} = 1000\text{ m}^2/\text{min}$

Find $\frac{dr}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$1000 = 2\pi(80)\frac{dr}{dt}$

$\frac{1000}{2\pi(80)} = \frac{dr}{dt}$

$\frac{dr}{dt} \approx \boxed{1.989\text{ m/min}}$