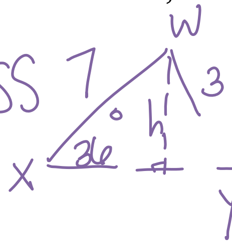
Precalculus **5.6 –The Law of Cosines and the Ambiguous Case**

Review: State how many triangles can be formed using the given information.

1) $\Delta XYW, x=3, y=7, m\angle X = 36°$

ASS



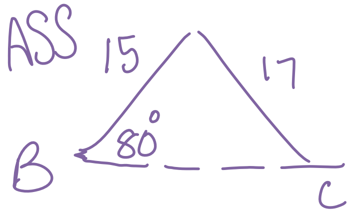
$h = 7\sin 36 = 4.1$

$\boxed{\text{No } \Delta}$

$h > x$

2) $\Delta ABC, b=17, c=15, m\angle B = 80°$

ASS

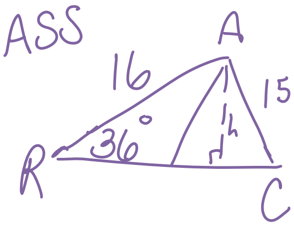


$\boxed{1\ \Delta}$

$b \geq c$

3) $\Delta CAR, c=16, r=15, m\angle R = 36°$

ASS

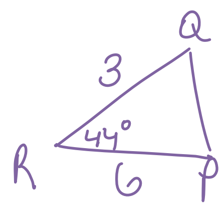


$h = 16\sin 36 = 9.4$

$h < r < c$

$\boxed{2\ \Delta s}$

4) $\Delta PQR, p=3, q=6, m\angle R = 44°$



SAS - $\boxed{1\ \Delta}$

When you know 2 sides and a non-included angle of a triangle, you may have __0__, __1__, or __2__ triangles. An easy way to see which situation you have, and to solve the resulting triangles, is to use __Law of Cosines__.

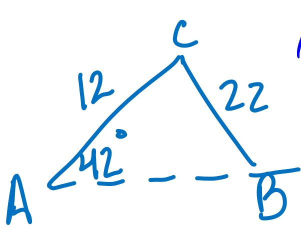
Reminder: Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Can also graph Quadratic and find zeros.

Examples: Determine how many triangles can be created, and solve each of them.

5) In ΔABC, a = 22, b = 12, $m\angle A = 42°$



ASS - Cosines

$22^2 = 12^2 + c^2 - 2(12)(c)\cos 42$

$0 = c^2 - 24\cos 42\, c - 340$

$c = -11.56, 29.40$ One Δ

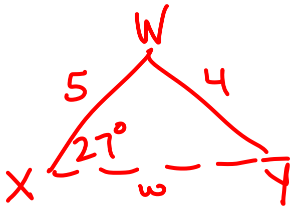
$\frac{\sin 42}{22} = \frac{\sin \angle B}{12}$

$\angle C$ is biggest

$\angle B = 21.4°$

$\angle C = 116.6°$

6) In ΔXYW, x = 4, y = 5, $m\angle X = 27°$



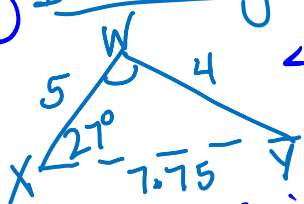
ASS- Cosines

$4^2 = 5^2 + w^2 - 2(5)(w)\cos 27°$

$0 = w^2 - (10\cos 27)w + 9$

$w = 7.75, 1.16$ ← both positive - 2 Δs!

① 1st Triangle



$\angle W$ is biggest

Use Law of Sines to find $\angle Y$:

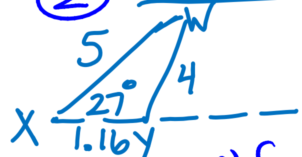
$$\frac{\sin 27}{4} = \frac{\sin Y}{5}$$

$\sin^{-1}$ $\angle Y = 34.6°$

Subtract from 180° to find W:

$\angle W = 118.4°$

② 2nd Triangle



$\angle W$ is smallest

Use Law of Sines to find $\angle W$:

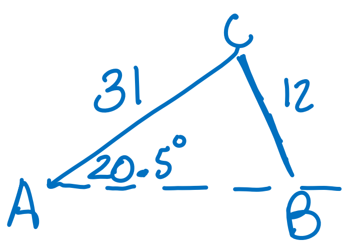
$$\frac{\sin 27}{4} = \frac{\sin W}{1.16}$$

$\angle W = 7.6°$

Subtract from 180° to find $\angle Y$:

$\angle Y = 145.4°$

7) In ΔABC, a = 12, b = 31, $m\angle A = 20.5°$



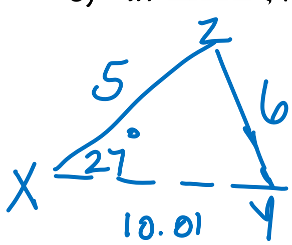
$12^2 = 31^2 + c^2 - 2(31)(c)\cos 20.5$

$0 = c^2 - 62\cos 20.5\, c + 817$

$c = -6.78, -.35$

No Δ possible!

8) In ΔXYZ, x = 6, y = 5, and $m\angle X = 27°$.



$6^2 = 5^2 + z^2 - 2(5)(z)\cos 27°$

$0 = z^2 - 10\cos 27\, z - 11$

$z = -1.10, 10.01$ $\boxed{1\ \Delta}$

$\angle Z$ is largest - find $\angle Y$ using Law of Sines: $\frac{\sin 27°}{6} = \frac{\sin Y}{5}$

$\angle Y = 22.2°$

Subtract from 180° to find $\angle Z$: $\angle Z = 130.8°$