Section 5.6 Exercises

1. Given: $B = 131°$, $c = 8$, $a = 13$ — an SAS case.

$$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{369.460} \approx 19.2;$$
$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.949) \approx 18.3°;$$
$$A = 180° - (B + C) \approx 30.7°.$$

2. Given: $C = 42°$, $b = 12$, $a = 14$ — an SAS case.

$$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx \sqrt{90.303} \approx 9.5;$$
$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.167) \approx 80.3°;$$
$$B = 180° - (A + C) \approx 57.7°.$$

3. Given: $a = 27$, $b = 19$, $c = 24$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.228) \approx 76.8°;$$
$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.728) \approx 43.2°;$$
$$C = 180° - (A + B) \approx 60°.$$

4. Given: $a = 28$, $b = 35$, $c = 17$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.613) \approx 52.2°;$$
$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.159) \approx 99.2°;$$
$$C = 180° - (A + B) \approx 28.6°.$$

5. Given: $A = 55°$, $b = 12$, $c = 7$ — an SAS case.

$$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{96.639} \approx 9.8;$$
$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.011) \approx 89.3°;$$
$$C = 180° - (A + B) \approx 35.7°.$$

6. Given: $B = 35°$, $a = 43$, $c = 19$ — an SAS case.

$$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{871.505} \approx 29.5;$$
$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.929) \approx 21.7°;$$
$$A = 180° - (B + C) \approx 123.3°.$$

7. Given: $a = 12$, $b = 21$, $C = 95°$ — an SAS case.

$$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx \sqrt{628.926} \approx 25.1;$$
$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.879) \approx 28.5°;$$
$$B = 180° - (A + C) \approx 56.5°.$$

8. Given: $b = 22$, $c = 31$, $A = 82°$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{1255.167} \approx 35.4$;
$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.788) \approx 37.9°$;
$C = 180° - (A + B) \approx 60.1°$.

9. No triangles possible ($a + c = b$).

10. No triangles possible ($a + b < c$).

11. Given: $a = 3.2$, $b = 7.6$, $c = 6.4$ — an SSS case.
$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.909) \approx 24.6°$;
$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.160) \approx 99.2°$;
$C = 180° - (A + B) \approx 56.2°$.

12. No triangles possible ($a + b < c$).

Exercises #13–16 are SSA cases, and can be solved with either the law of sines or the law of cosines. The law of cosines solution is shown.

13. Given: $A = 42°$, $a = 7$, $b = 10$ — an SSA case. Solve the quadratic equation $7^2 = 10^2 + c^2 - 2(10)c\cos 42°$, or $c^2 - (14.862\ldots)c + 51 = 0$; there are two positive solutions: ≈ 9.487 *or* 5.376. Since $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$:
$c_1 \approx 9.5$, $B_1 \approx \cos^{-1}(0.294) \approx 72.9°$, and
$C_1 = 180° - (A + B_1) \approx 65.1°$,
or
$c_2 \approx 5.4$, $B_2 \approx \cos^{-1}(-0.294) \approx 107.1°$, and
$C_2 = 180° - (A + B_2) \approx 30.9°$.

14. Given: $A = 57°$, $a = 11$, $b = 10$ — an SSA case. Solve the quadratic equation $11^2 = 10^2 + c^2 - 2(10)C\cos 57°$, or $c^2 - (10.893)c - 21 = 0$; there is one positive solution $c = 12.6$. Since $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$,
$B \approx \cos^{-1}(0.647) \approx 49.7°$ and $C = 180° - (A + B) \approx 73.3°$.

15. Given: $A = 63°$, $a = 8.6$, $b = 11.1$ — an SSA case. Solve the quadratic equation
$8.6^2 = 11.1^2 + c^2 - 2(11.1)c\cos 63°$, or
$c^2 - (10.079)c + 49.25 = 0$; there are no real solutions, so there is no triangle.

16. Given: $A = 71°$, $a = 9.3$, $b = 8.5$ — an SSA case. Solve the quadratic equation
$9.3^2 = 8.5^2 + c^2 - 2(8.5)c\cos 71°$, or
$c^2 - (5.535)c - 14.24 = 0$; there is one positive solution: $c \approx 7.4$. Since $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$,
$B \approx \cos^{-1}(0.503) \approx 59.8°$ and $C = 180° - (A + B) \approx 49.2°$.

17. Given: $A = 47°$, $b = 32$, $c = 19$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{555.689} \approx 23.573$,
so Area $\approx \sqrt{49431.307} \approx 222.33$ ft.^{2} (using Heron's formula). Or, use Area $= \frac{1}{2}bc\sin A = \frac{1}{2}(32)(19)$ $\sin 47° \approx 222.33$ ft^2.

18. Given: $A = 52°$, $b = 14$, $c = 21$ — an SAS case.
$a = \sqrt{b^2 + c^2 - 2bc\cos A} \approx \sqrt{274.991} \approx 16.583$,
so Area $\approx \sqrt{13418.345} \approx 115.84$ m.^{2} (using Heron's formula). Or, use Area $= \frac{1}{2}bc\sin A = \frac{1}{2}(14)(21)$ $\sin 52° \approx 115.84$ m^2.

19. Given: $B = 101°$, $a = 10$, $c = 22$ — an SAS case.
$b = \sqrt{a^2 + c^2 - 2ac\cos B} \approx \sqrt{667.955} \approx 25.845$,
so Area $\approx \sqrt{11659.462} \approx 107.98$ cm.^{2} (using Heron's formula). Or, use Area $= \frac{1}{2}ac\sin B = \frac{1}{2}(10)(22)$ $\sin 101° \approx 107.98$ cm^2.

20. Given: $C = 112°$, $a = 1.8$, $b = 5.1$ — an SAS case.
$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx \sqrt{36.128} \approx 6.011$,
so Area $\approx \sqrt{18.111} \approx 4.26$ in.^{2} (using Heron's formula). Or, use Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(1.8)(5.1)$ $\sin 112° \approx 4.26$ in^2.

For #21–28, a triangle can be formed if $a + b < c$, $a + c < b$, and $b + c < a$.

21. $s = \frac{17}{2}$; Area $= \sqrt{66.9375} \approx 8.18$.

22. $s = \frac{21}{2}$; Area $= \sqrt{303.1875} \approx 17.41$.

23. No triangle is formed ($a + b = c$).

24. $s = 27$; Area $= \sqrt{12{,}960} = 36\sqrt{10} \approx 113.84$.

25. $a = 36.4$; Area $= \sqrt{46{,}720.3464} \approx 216.15$.

26. No triangle is formed ($a + b < c$).

27. $s = 42.1$; Area $= \sqrt{98{,}629.1856} \approx 314.05$.

28. $s = 23.8$; Area $\sqrt{10{,}269.224} \approx 101.34$.

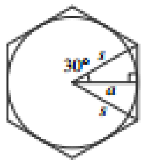
29. Let $a = 4$, $b = 5$, and $c = 6$. The largest angle is opposite the largest side, so we call it C. Since $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, $C = \cos^{-1}\left(\frac{1}{8}\right) \approx 82.819°$ ≈ 1.445 radians.

30. The shorter diagonal splits the parallelogram into two (congruent) triangles with $a = 26$, $B = 39°$, and $c = 18$. The diagonal has length $b = \sqrt{a^2 + c^2 - 2ac\cos B}$ $\approx \sqrt{272.591} \approx 16.5$ ft.

31. Following the method of Example 3, divide the hexagon into six triangles. Each has two 12-inch sides that form a 60° angle.
$6 \times \frac{1}{2}(12)(12)\sin 60° = 216\sqrt{3} \approx 374.1$ square inches.

32. Following the method of Example 3, divide the nonagon into nine triangles. Each has two 10-inch sides that form a 40° angle.
$9 \times \frac{1}{2}(10)(10)\sin 40° \approx 289.3$ square inches.

33.

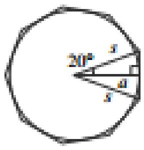


In the figure, $a = 12$ and so $s = 12 \sec 30° = 8\sqrt{3}$.
The area of the hexagon is

$6 \times \frac{1}{2}(8\sqrt{3})(8\sqrt{3}) \sin 60° = 288\sqrt{3}$

≈ 498.8 square inches.

34.



In the figure, $a = 10$ and so $s = 10 \sec 20°$. The area of the nonagon is

$9 \times \frac{1}{2}(10 \sec 20°)(10 \sec 20°) \sin 40° \approx 327.6$ square inches.

35. Given: $C = 54°$, $BC = a = 160$, $AC = b = 110$ — an SAS case. $AB = c = \sqrt{a^2 + b^2 - 2ab \cos C}$
$\approx \sqrt{17{,}009.959} \approx 130.42$ ft.

36. **(a)** The home-to-second segment is the hypotenuse of a right triangle, so the distance from the pitcher's rubber to second base is $90\sqrt{2} - 60.5 \approx 66.8$ ft. This is a bit more than
$c = \sqrt{60.5^2 + 90^2 - 2(60.5)(90) \cos 45°}$
$\approx \sqrt{4059.857} \approx 63.7$ ft.

(b) $B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.049)$
$\approx 92.8°$.

37. **(a)** $c = \sqrt{40^2 + 60^2 - 2(40)(60) \cos 45°}$
$\approx \sqrt{1805.887} \approx 42.5$ ft.

(b) The home-to-second segment is the hypotenuse of a right triangle, so the distance from the pitcher's rubber to second base is $60\sqrt{2} - 40 \approx 44.9$ ft.

(c) $B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.057)$
$\approx 93.3°$.

38. Given: $a = 175$, $b = 860$, and $C = 78°$. An SAS case, so
$AB = c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{707{,}643.581}$
≈ 841.2 ft.

39. **(a)** Using right ΔACE, $m\angle CAE = \tan^{-1}\left(\frac{6}{18}\right)$
$= \tan^{-1}\left(\frac{1}{3}\right) \approx 18.4°$.

(b) Using $A \approx 18.435°$, we have an SAS case, so
$DF = \sqrt{9^2 + 12^2 - 2(9)(12) \cos A} \approx \sqrt{20.084}$
≈ 4.5 ft.

(c) $EF = \sqrt{18^2 + 12^2 - 2(18)(12) \cos A} \approx \sqrt{58.168}$
≈ 7.6 ft.

40. After two hours, the planes have traveled 700 and 760 miles, and the angle between them is 22.5°, so the distance is $\sqrt{700^2 + 760^2 - 2(700)(760) \cos 22.5°}$
$\approx \sqrt{84{,}592.177} \approx 290.8$ mi.

41. $AB = \sqrt{73^2 + 65^2 - 2(73)(65) \cos 8°}$
$\approx \sqrt{156.356} \approx 12.5$ yd.

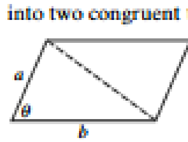
42. $m\angle HAB = 135°$, so
$HB = \sqrt{20^2 + 20^2 - 2(20)(20) \cos 135°}$
$\approx \sqrt{1365.685} \approx 37.0$ ft.
Note that $\overline{AB}$ is the hypotenuse of an equilateral right triangle with leg length $\frac{20}{\sqrt{2}} = 10\sqrt{2}$, and $\overline{HC}$ is the hypotenuse of an equilateral right triangle with leg length $20 + 10\sqrt{2}$, so $HC = \sqrt{2(20 + 10\sqrt{2})^2} \approx 48.3$ ft.
Finally, using right ΔHAD with leg lengths $HA = 20$ ft and $AD = HC \approx 48.3$ ft, we have
$HD = \sqrt{HA^2 + AD^2} \approx 52.3$ ft.

43. $AB = c = \sqrt{2^2 + 3^2} = \sqrt{13}$, $AC = b = \sqrt{1^2 + 3^2}$
$= \sqrt{10}$, and $BC = a = \sqrt{1^2 + 2^2} = \sqrt{5}$, so
$m\angle CAB = A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$
$= \cos^{-1}\left(\frac{9}{\sqrt{130}}\right) \approx 37.9°$.

44. ΔABC is a right triangle ($C = 90°$), with $BC = a$
$= \sqrt{2^2 + 2^2} = 2\sqrt{2}$ and $AC = b = 1$, so $AB = c$
$= \sqrt{a^2 + b^2} = 3$ and $B = m\angle ABC = \sin^{-1}\left(\frac{1}{3}\right)$
$\approx 19.5°$.

45. True. By the law of cosines, $b^2 + c^2 - 2bc \cos A = a^2$, which is a positive number. Since $b^2 + c^2 - 2bc \cos A > 0$, it follows that $b^2 + c^2 > 2bc \cos A$.

46. True. The diagonal opposite angle θ splits the parallelogram into two congruent triangles, each with area $\frac{1}{2}ab \sin \theta$.



47. Following the method of Example 3, divide the dodecagon into 12 triangles. Each has two 12-inch sides that form a 30° angle.

$12 \times \frac{1}{2}(12)(12) \sin 30° = 432$

The answer is B.

48. The semiperimeter is $s = (7 + 8 + 9)/2 = 12$. Then by Heron's formula, $A = \sqrt{12(12 - 7)(12 - 8)(12 - 9)}$
$= 12\sqrt{5}$. The answer is B.

49. After 30 minutes, the first boat has traveled 12 miles and the second has traveled 16 miles. By the law of cosines, the two boats are $\sqrt{12^2 + 16^2 - 2(12)(16) \cos 110°} \approx 23.05$ miles apart. The answer is C.