Tuesday, November 21, 2016

Notes - Estimating Roots w/Linearization, Differentials

HW Questions

5.5 Estimating Roots

Use Linearization to Estimate a root.

(1) $\sqrt{101}$ $a=100$ $f(x)=\sqrt{x}$ > write linearization at $a=100$

point: $(100, 10)$ $f(100)=\sqrt{100}=10$

slope: $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$

$f'(100)=\frac{1}{2\sqrt{100}}=\frac{1}{20}$

tangent line $y-10=\frac{1}{20}(x-100)$

linearization $L(x)=\frac{1}{20}(x-100)+10$

$L(101)=\frac{1}{20}(101-100)+10=10\frac{1}{20}=10.05$

How accurate? $\sqrt{101}=10.04988$

Error $=|10.05-10.04988|=1.24\times10^{-4}$

Error less than 10^{-3}

(2) $\sqrt[3]{26}$ $a=27$ $f(x)=\sqrt[3]{x}$

point $(27, 3)$

slope $=f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$

$f'(27)=\frac{1}{3}(27)^{-\frac{2}{3}}=\frac{1}{3}\cdot\frac{1}{9}=\frac{1}{27}$

tan line: $y-3=\frac{1}{27}(x-27)$

lin: $L(x)=\frac{1}{27}(x-27)+3$

$L(26)=\frac{1}{27}(26-27)+3 = 2\frac{26}{27} \approx 2.963$

actual value: $\sqrt[3]{26} \approx 2.962$

Error: 4.67×10^{-4} less than 10^{-3}

Differentials : Leibnitz Notation $\frac{dy}{dx}$

$dy \rightarrow$ differentials $\quad dx \rightarrow$ infinitely small increments in x- or y-direction

$$\frac{dy}{dx} = f'(x)$$

$$df = dy = f'(x)dx \qquad \left[\begin{array}{l} df=dy \\ \text{bc } f(x)=y \end{array}\right]$$

(1) Find dy if $y = x\ln x$, $x=1$ and $dx=.01$

$\frac{dy}{dx}=f'(x) \rightarrow dy = f'(x)\cdot dx$

$$\boxed{dy=(1+\ln x)dx}$$

at $x=1$, $dx=.01$ $\quad dy=(1+\ln 1)(.01)$

$=(1+0)(.01)$

$dy=.01$