

Section 5.4C Notes

Fundamental Theorem Part 1 Graph Problems

Let  $F(x) = \int_0^x f(t)dt$  where  $f$  is the function graphed below (all segments and a semicircle).

a) Evaluate  $F(-2)$ ,  $F(0)$ ,  $F(2)$ , and  $F(7)$

$$F(-2) = \int_0^{-2} f(t)dt = - \int_{-2}^0 f(t)dt = -\left(-\frac{1}{4}\pi \cdot 2^2\right) = \boxed{\pi}$$

$$F(0) = \int_0^0 f(t)dt = \boxed{0}$$

$$F(2) = \int_0^2 f(t)dt = \frac{1}{2} \cdot 2 \cdot 2 = \boxed{2}$$

$$F(7) = \int_0^7 f(t)dt = \frac{1}{2} \cdot 4 \cdot 2 - \frac{1}{2} \cdot 3 \cdot 3 = \frac{8}{2} - \frac{9}{2} = \boxed{-\frac{1}{2}}$$

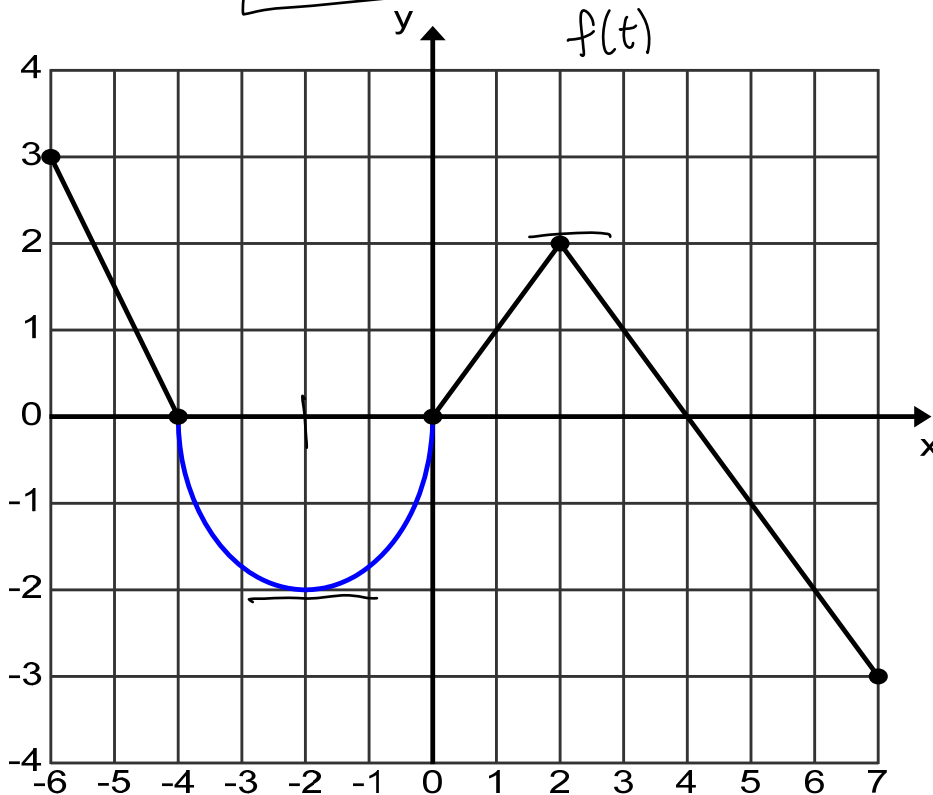
b) Identify all the critical points of  $F$  in the interval  $[-6,7]$ .

$$F'(x) = f(x) = 0 \leftarrow \text{and sign change } \boxed{x = -4, 0, 4}$$

c) Identify all  $x$ -coordinates of the inflection points of  $F$  in the interval  $[-6,7]$ .

$$F''(x) = f'(x) = 0 \text{ or } \emptyset \leftarrow \text{add slope change signs}$$

$$\boxed{x = -2, x = 2}$$



Let  $F(x) = \int_0^x f(t) dt$  where  $f$  is the function graphed below (all segments and a semicircle).

a) Evaluate  $F(-8)$ ,  $F(-3)$ ,  $F(0)$ ,  $F(5)$ , and  $F(6)$

$$F(-8) = \int_0^{-8} f(t) dt = - \int_{-8}^0 f(t) dt = - \left( \frac{1}{2}(2+4) \cdot 2 + \frac{1}{2} \cdot 3 \cdot 4 + \frac{1}{4} \pi \cdot 3^2 \right)$$

$$= - \left( 6 + 6 + \frac{9}{4} \pi \right) = \boxed{-12 - \frac{9}{4} \pi}$$

$$F(-3) = - \int_{-3}^0 f(t) dt = \boxed{-\frac{9}{4} \pi}$$

$$F(5) = \int_0^5 f(t) dt = \frac{1}{4} \pi \cdot 3^2 - \frac{1}{2} \cdot 2 \cdot 3$$

$$= \boxed{\frac{9}{4} \pi - 3}$$

$$F(0) = \int_0^0 f(t) dt = \boxed{0}$$

$$F(6) = \int_0^6 f(t) dt = \frac{9}{4} \pi - 3 - \frac{1}{2}(2+3) \cdot 1$$

$$= \boxed{\frac{9}{4} \pi - \frac{11}{2}}$$

b) Identify all the critical points of  $F$  in the interval  $[-8, 6]$ .

$$F'(x) = f(x) = 0 \leftarrow \text{and sign change}$$

$$\boxed{x=3}$$

c) Identify all  $x$ -coordinates of the inflection points of  $F$  in the interval  $[-8, 6]$ .

$$F''(x) = f'(x) = 0 \text{ or } \phi \leftarrow \text{and slopes change sign}$$

$$\boxed{x=-6, x=-3, x=0, x=5}$$

