(1) $f(x) = (x-2)^5(x+3)^4$

$f'(x) = 5(x-2)^4(x+3)^4 + 4(x+3)^3(x-2)^5 = 0$ crit pts

[GCF] $(x+3)^3(x-2)^4[5(x+3) + 4(x-2)] = 0$

$x = -3, 2$ $\quad$ $5x + 15 + 4x - 8 = 0$

$9x = -7$

$x = \frac{-7}{9}$ $\quad$ $\boxed{C}$

(2) $f(x) = (x-2)(x-3)^2$

$f'(x) = (x-3)^2 + 2(x-3)(x-2)$

$= x^2 - 6x + 9 + 2x^2 - 10x + 12$

$f'(x) = 3x^2 - 16x + 21 = 0$

$= (3x - 7)(x - 3) = 0$

$x = \frac{7}{3}, 3$

$f''(x) = 6x - 16$ $\quad$ $f''(\frac{7}{3}) = 6(\frac{7}{3}) - 16 = -2$ Neg *

$f''(3) = 6(3) - 16 = 2$ Pos

$x = \frac{7}{3}$ is rel max $\boxed{D}$

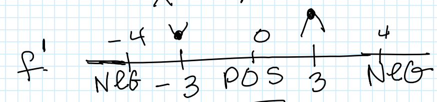
(3) $g(x) < 0$

negative

$f'(x) = (x^2 - 9)g(x)$

$x^2 - 9 = 0$ $\quad$ $(x+3)(x-3)(neg)$

$x = 3, -3$



$\boxed{B}$

(4) $f(x) = 3\ln(x^2+2) - 2x$ $\quad$ $[-2, 4]$

(a) $f'(x) = \frac{3}{x^2+2} \cdot 2x - 2$

$f'(x) = \frac{6x}{x^2+2} - 2$ Never undefined

$\frac{6x}{x^2+2} - 2 = 0$

$\frac{6x}{x^2+2} = 2$

$6x = 2x^2 + 4$

$2x^2 - 6x + 4 = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

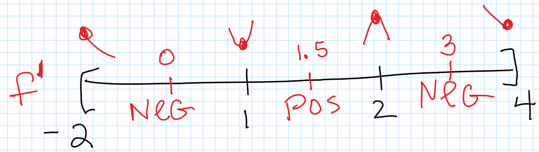
$x = 1, 2$

Endpoints $f(-2) = 3\ln((-2)^2+2) - 2(-2)$

$= 3\ln 6 + 4 \approx 6.908$

$f(4) = 3\ln(4^2+2) - 2(4)$

$= 3\ln 18 - 8 \approx .671$



$f(1) = 3\ln((1)^2+2) - 2(1)$

$= 3\ln 3 - 2 \approx 1.296$

$f(2) = 3\ln(2^2+2) - 2(2)$

$= 3\ln 6 - 4 \approx 1.375$

$x = -2$ is location of rel. max bc it is left endpoint and f' is neg to the right
$x = 1$ is location of rel min bc f' changes from $-$ to $+$
$x = 2$ is location of rel max bc f' changes from $+$ to $-$
$x = 4$ is location of rel min bc it is right endpt and f' is neg to the left on endpt.

(b) $f''(x) = \frac{(x^2+2)6 - 6x(2x)}{(x^2+2)} = \frac{6x^2 + 12 - 12x^2}{x^2+2} = \frac{-6x^2+12}{x^2+2} = \frac{-6(x^2-2)}{x^2+2}$

Quotient Rule

$f''(x)$ ... never undefined

$f''(x) = 0$ at $x = \pm\sqrt{2}$

f'': -10 NEG -2 POS $\sqrt{2}$ NEG

$\boxed{f(x) \text{ has POI @ } x = \pm\sqrt{2} \text{ bc } f''(x) \text{ changes sign @ } \pm\sqrt{2}}$

(c) $3\ln 6 + 4 \approx 6.908$ is abs max bc it is max w/ highest value.

(14) $s = -16t^2 + 96t + 112$

(a) $v(0) = 96$ ft/sec

(b) Max ht will occur when $v(t) = 0$

$v(t) = -32t + 96 = 0$

$t = 3$

$s' = v(t)$: 0 POS 3 NEG (when it hits ground)

max at $t = 3$

max ht is $\boxed{s(3) = 256 \text{ ft}}$

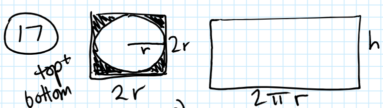
(c) $s = 0 = -16t^2 + 96t + 112$

$0 = -16(t^2 - 6t - 7)$

$0 = -16(t+1)(t-7)$

$t = 7$ seconds

$v(7) = -32(7) + 96 = \boxed{-128 \text{ ft/s}}$

(17) 

$V = 1000 \text{ cm}^3 = \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

$A = 2(4r^2) + 2\pi r h$

$= 8r^2 + 2\pi r h$

$A = 8r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$A = 8r^2 + \frac{2000}{r}$

$\frac{dA}{dr} = 16r - \frac{2000}{r^2} = 0$

$16r = \frac{2000}{r^2}$

$16r^3 = 2000$

$r^3 = 125$

$r = 5$

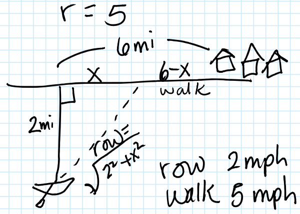
A': 0 NEG 5 POS $\sqrt{\frac{1000}{\pi}}$

$r = 5$ is min

$h = \frac{1000}{\pi(25)} = \frac{40}{\pi}$

Most economical ratio is:

$\frac{h}{r} = \frac{40}{\pi}$

(20) 

row 2 mph

walk 5 mph

$\text{time (hours)} = \frac{\text{miles}}{\text{mile/hour}}$ ✓

Total Time $= \frac{\sqrt{4+x^2}}{2} + \frac{6-x}{5}$

$\frac{dT}{dx} = \frac{1}{2}\left(\frac{1}{2}\right)(4+x^2)^{-1/2}(2x) - \frac{1}{5}$

$0 = \frac{x}{2\sqrt{4+x^2}} - \frac{1}{5}$

$\frac{1}{5} = \frac{x}{2\sqrt{4+x^2}}$

$2\sqrt{4+x^2} = 5x$

$4(4+x^2) = 25x^2$

$16 + 4x^2 = 25x^2$

$16 = 21x^2$

$\pm\sqrt{16} = x$

why are endpts included?

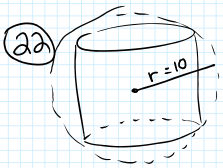
T': 0 (local max) NEG $\sqrt{\frac{16}{21}}$ (min) POS 6 (local max)

x can't be neg

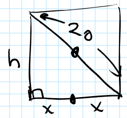
$x = \sqrt{\frac{16}{21}}$ minimizes time,

$\boxed{\text{So } \sqrt{\tfrac{16}{21}} \approx .87 \text{ miles downshore from nearest pt to her boat}}$

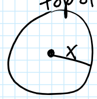
(22)



sideview of cylinder



top of cylinder



max Vol of Cylinder

$V = \pi r^2 h$

$V = \pi x^2 \sqrt{400-4x^2}$

$\pi x^2 (400-4x^2)^{\frac{1}{2}}$

$\frac{dV}{dx} = \pi\left[2x(400-4x^2)^{\frac{1}{2}} + x^2\left(\frac{1}{2}\right)(400-4x^2)^{-\frac{1}{2}}(-8x)\right]$

$= \pi\left[2x\sqrt{400-4x^2} - \frac{4x^3}{\sqrt{400-4x^2}}\right]$

$h^2 + (2x)^2 = 400$

$h = \sqrt{400-4x^2}$

$400 - 4x^2 \geq 0$

$x \leq 10$ & $x \geq -10$

$[-10, 10]$

$x > 0$, so $(0, 10)$

$h < 10$

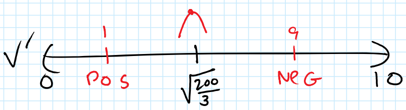
$2x\sqrt{400-4x^2} = \frac{4x^3}{\sqrt{400-4x^2}}$

$2x(400-4x^2) = 4x^3$ $\quad x \neq 0$ so

$400 - 4x^2 = 2x^2$

$400 = 6x^2$

$\pm\sqrt{\frac{200}{3}} = x$



$x = \sqrt{\frac{200}{3}}$ is max $\quad h = \sqrt{400 - 4\left(\frac{200}{3}\right)}$

$h = \sqrt{400 - \frac{800}{3}} = \sqrt{\frac{400}{3}}$

Max Vol $= \pi\left(\frac{200}{3}\right)\sqrt{\frac{400}{3}} = \boxed{\frac{4000\pi}{3\sqrt{3}} \text{ cm}^3}$