Monday, March 6 - Late Start

- ✧ 5.3 Sum & Difference Identities
- ✧ Practice

PLAN

(P+L)(A+N)

PA+PN+LA+LN

Your plan has been foiled

5.3 Sum + Difference Identities

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\sin a \cos b \pm \cos a \sin b}{\cos a \cos b \mp \sin a \sin b}$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Examples:

(1) $\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ (a − b)

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

(2) $\sin 285^\circ = \sin(225 + 60^\circ)$ (a + b)

$= \sin 225^\circ \cos 60^\circ + \cos 225^\circ \sin 60^\circ$

$= -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

(3) $\sin \frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

(4) $\tan\frac{5\pi}{12} = \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}\cdot 1}\ \frac{3}{3} = \boxed{\frac{\sqrt{3}+3}{3-\sqrt{3}}}$$