Pre-Calculus
Practice 5.3-5.4

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Part I: Use a sum or difference identity to find an exact value for:

1) $\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$= \sin 60 \cos 45 + \cos 60 \sin 45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2) $\cos 195^\circ = \cos(150^\circ + 45^\circ)$ $\quad 135^\circ + 60^\circ$

$= \cos 150 \cos 45 - \sin 150 \sin 45^\circ$ $\quad \cos 135 \cos 60 - \sin 135 \sin 60$

$= -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

3) $\tan(-15^\circ)$

$= \tan(30-45)$

$= \frac{\tan 30 - \tan 45}{1 + \tan 30 \tan 45}$

$= \frac{\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3}(1)} = \frac{\frac{\sqrt{3}-3}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{\sqrt{3}-3}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}} = \boxed{\frac{\sqrt{3}-3}{3+\sqrt{3}}}$

OR $\boxed{\frac{1-\sqrt{3}}{\sqrt{3}+1}}$

Part II: Prove each of the following:

1) $\sin(x-\pi) = -\sin x$

$\sin x \cos \pi - \cos x \sin \pi =$

$\sin x(-1) - \cancel{\cos x(0)} =$

$-\sin x$ ✓

2) $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

$\cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2} =$

$\cancel{\cos x(0)} + \sin x(1) =$

$\sin x$ ✓

3) $\tan\left(x + \frac{\pi}{4}\right) = \frac{1+\tan x}{1-\tan x}$

$\frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}} =$

$\frac{\tan x + 1}{1 - \tan x(1)} =$

$\frac{1+\tan x}{1-\tan x}$ ✓

4) $\cos 2x = 1 - 2\sin^2 x$

$\cos(x+x) =$

$\cos x \cos x - \sin x \sin x$

$\cos^2 x - \sin^2 x =$

$1 - \sin^2 x - \sin^2 x =$

$1 - 2\sin^2 x$ ✓

Part III: Using your double-angle identities, fill in the missing information.

1) $\cos 350^\circ = 1 - 2\,\underline{\sin^2 175^\circ}$

$u = 175^\circ$

2) $\sin(10x) = 2\,\underline{\sin 5x \cos 5x}$

$u = 5x$

3) $\cos(14x) = \underline{\cos^2 7x} - \sin^2 7x$

$u = 7x$

4) $\tan 250^\circ = \underline{\dfrac{2\tan 125^\circ}{1-\tan^2 125^\circ}}$

$u = 125^\circ$

Part IV: Find all solutions to each equation in the interval $[0, 2\pi)$

1) $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2} \quad \cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

2) $\sin 2x = 2\sin x$

$2\sin x \cos x = 2\sin x$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = 1$

$\boxed{x = 0, \pi}$

3) $\cot x - \sin 2x = 0$

$\cot x - 2\sin x \cos x = 0$

$\frac{\cos x}{\sin x} - \frac{2\sin^2 x \cos x}{\sin x} = 0$

$\frac{\cos x - 2\sin^2 x \cos x}{\sin x} = 0$

$\frac{\cos x(1 - 2\sin^2 x)}{\sin x} = 0$

$(\cot x)(1 - 2\sin^2 x) = 0$

$\cot x = 0 \quad \sin^2 x = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$