



Friday, October 28, 2016

Opener - #1 on Green Sheet

HW Check & Questions

5.2 - Mean Value Theorem

$$f(x) = x\sqrt{16-x^2} \quad \text{Domain: } [-4, 4]$$

$$f'(x) = \sqrt{16-x^2} - \frac{x^2}{\sqrt{16-x^2}}$$

$$\sqrt{16-x^2} - \frac{x^2}{\sqrt{16-x^2}} = 0$$

$$\sqrt{16-x^2} \cdot \sqrt{16-x^2} = \frac{x^2}{\sqrt{16-x^2}} \cdot \sqrt{16-x^2}$$

$$16-x^2 = x^2$$

$$16 = 2x^2$$

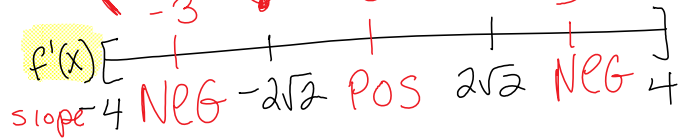
$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

Critical pts:

$$f' \text{ und: } -4, 4$$

$$f'(x) = 0 \quad \begin{matrix} \text{max} & & \text{min} & & \text{max} & & \text{min} \\ \uparrow & & \downarrow & & \uparrow & & \downarrow \end{matrix}$$



x	f(x)	
-4	0	Local Max
$-2\sqrt{2}$	-8	Abs Min
$2\sqrt{2}$	8	Abs Max
4	0	Local Min

5.2 - Mean Value Theorem

Think about this: A person drives on a tollway. They enter at 4:00 pm and exit at 5:00 pm, having driven 85 miles. The speed limit on the tollway is 65 mph. Although no cop saw them speeding, they receive a ticket in the mail. Is this justified?

Mean value Theorem

If a function is continuous on $[a, b]$ and differentiable on (a, b) then

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$b - a$$

average slope between endpoints is found
at some point in interval.

Example :

$$\textcircled{1} \quad g(x) = 4x^3 - x^2 + 4 \quad [-1, 1]$$

\textcircled{A} Does MVT apply?

- cont. on $[-1, 1]$ ✓
diff. on $[-1, 1]$ ✓ Yes.

$$\textcircled{B} \quad \frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{f(1) - f(-1)}{1 - (-1)} = f'(c)$$

$$\frac{7 - (-1)}{2} = 12c^2 - 2c$$

$$\frac{8}{2} =$$

avg slope \rightarrow

$$4 = 12c^2 - 2c \leftarrow \text{derivative}$$

$$0 = 12c^2 - 2c - 4$$

$$0 = 6c^2 - c - 2$$

$$0 = (3c - 2)(c + 1)$$

$$c = \frac{2}{3}, -\frac{1}{2}$$

