

5.1 - 5.2 Review WS Solutions

Note Title

$$\textcircled{1} \sin x \sec x = \sin x \cdot \frac{1}{\cos x} = \boxed{\tan x}$$

$$\textcircled{2} \sin^2 x (\cot^2 x + 1) = \sin^2 x (\csc^2 x) = \boxed{1}$$

$$\textcircled{3} \frac{\csc x}{\cot x} = \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$$

$$\begin{aligned} \textcircled{14} \quad & 1 - \frac{\sin^2 x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \\ &= 1 - \frac{\sin^2 x (1 + \cos x)}{1 - \cos^2 x} \\ &= 1 - \frac{\sin^2 x (1 + \cos x)}{\sin^2 x} \\ &= 1 - 1 - \cos x \\ &= \boxed{-\cos x} \end{aligned}$$

$$\begin{aligned} \text{OR} \quad & \frac{1 - \cos x}{1 - \cos x} - \frac{\sin^2 x}{1 - \cos x} \\ &= \frac{1 - \cos x - (1 - \cos^2 x)}{1 - \cos x} \\ &= \frac{-\cos x + \cos^2 x}{1 - \cos x} \\ &= \frac{-\cos x (1 - \cos x)}{1 - \cos x} \\ &= \boxed{-\cos x} \end{aligned}$$

$$\textcircled{5} \sec^2 x + \csc^2 x$$

$$\begin{aligned} & \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} \\ &= \frac{1}{\cos^2 x \sin^2 x} \\ &= \boxed{\sec^2 x \csc^2 x} \end{aligned}$$

$$\textcircled{6} \quad \csc x - \cos x \cot x = \sin x$$

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} =$$

$$\frac{1 - \cos^2 x}{\sin x} =$$

$$\frac{\sin^2 x}{\sin x} = \sin x \quad \checkmark$$

$$\textcircled{7} \quad \frac{\sec x + \tan x}{\csc x + 1} = \tan x$$

$$\frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\sin x} + 1} =$$

$$\frac{\frac{1 + \sin x}{\cos x}}{\frac{1 + \sin x}{\sin x}} =$$

$$\frac{\cancel{1 + \sin x}}{\cos x} \cdot \frac{\sin x}{\cancel{1 + \sin x}} =$$

$$\frac{\sin x}{\cos x} =$$

$$\tan x \quad \checkmark$$

$$\textcircled{8} \tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x =$$

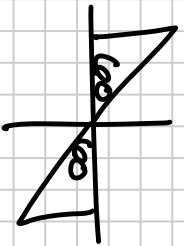
$$\frac{\sin^2 x - \sin^2 \cos^2 x}{\cos^2 x} =$$

$$\frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} =$$

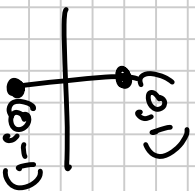
$$\tan^2 x \sin^2 x \checkmark$$

$$\textcircled{10} \tan x + \sqrt{3} = 0$$

$$\tan x = -\sqrt{3}$$



$$x = \frac{2\pi}{3}, \frac{5\pi}{3}$$



$$\textcircled{9} \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

$$\frac{\sin x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} =$$

$$\frac{\sin x (1 - \cos x)}{1 - \cos^2 x} =$$

$$\frac{\cancel{\sin x} (1 - \cos x)}{\cancel{\sin x} x} =$$

$$\frac{1 - \cos x}{\sin x} \checkmark$$

$$\textcircled{11} 2 \sin x \cos x = \sqrt{2} \cos x$$

$$2 \sin x \cos x - \sqrt{2} \cos x = 0$$

$$\cos x (2 \sin x - \sqrt{2}) = 0$$

$$\cos x = 0 \quad \sin x = \frac{\sqrt{2}}{2}$$

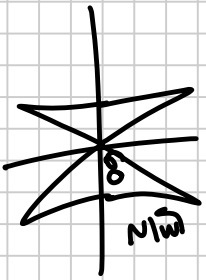
$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}$$



$$\textcircled{12} 4 \sin^2 x = 3$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$



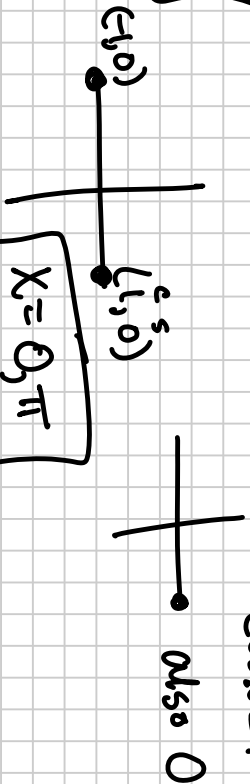
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{13} \tan x \sec x = \tan x$$

$$\tan x \sec x - \tan x = 0$$

$$\tan x (\sec x - 1) = 0$$

$$\tan x = 0 \quad \sec x = 1 \quad \cos x = 1$$



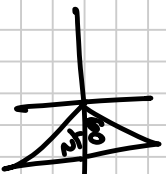
$$\textcircled{14} 2 \sec^2 x - 3 \sec x - 2 = 0$$

$$(2 \sec x + 1)(\sec x - 2) = 0$$

$$\sec x = -\frac{1}{2}$$

$$\cos x = -2$$

$$x = \phi$$



$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{15} \tan^2 x - \sec x - 1 = 0$$

$$\sec^2 x - 1 - \sec x - 1 = 0$$

$$\sec^2 x - \sec x - 2 = 0$$

$$(\sec x + 1)(\sec x - 2) = 0$$

$$\sec x = -1 \quad \sec x = 2$$

$$\cos x = -1 \quad \cos x = \frac{1}{2}$$

