Calc AB: 4.6 Related Rates examples

Work with your partner to setup and solve the following problems.

Spherical Balloon

positive — volume rate $\frac{dV}{dt}$

Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet. (refer to your notes for help)

$r = 2$ ft Find $\frac{dr}{dt}$ $\frac{dV}{dt} = 4.5$ ft^3/min

sphere:

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4.5 = 4\pi(2)^2 \frac{dr}{dt}$

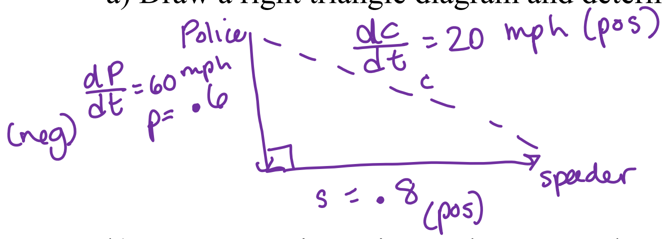
$\frac{dr}{dt} \approx$ ~~.179~~ ft/min .0895 ft/min

A Highway Chase

A police cruiser, approaching an intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph when they use the radar, then how fast is the speeder going? find $\frac{ds}{dt}$

pos when dist increasing
neg when dist decreasing

a) Draw a right triangle diagram and determine your givens:



$c^2 = .6^2 + .8^2$

$c = 1$

b) Set up equation using Pythagorean Theorem:

$c^2 = p^2 + s^2$

c) Take derivative w/respect to t of your equation (use the squared version to make it easier!)

$2c\frac{dc}{dt} = 2p\frac{dp}{dt} + 2s\frac{ds}{dt}$

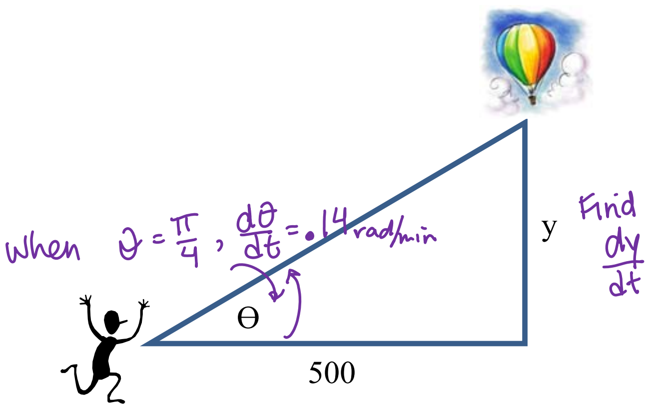
$2(1)(20) = 2(.6)(-60) + 2(.8)\frac{ds}{dt}$

d) Solve for rate (speed) of the "speeder" (be sure to use units):

$\frac{ds}{dt} = $ 70 mph

A Rising Balloon

A hot-air balloon rising up from a level field is tracked by a range finder 500 from the liftoff point. At the moment the range finder's angle of elevation is $\frac{\pi}{4}$, that angle is increasing at a rate of 0.14 rad/min. How fast is the balloon rising at that moment?



a) Label givens and diagram

$$\tan\theta = \frac{y}{500}$$

$$\tan\frac{\pi}{4} = \frac{y}{500}$$

$$y = 500$$

b) Write a tangent equation to solve.

$$\tan\theta = \frac{y}{500}$$

c) Find derivative of your equation with respect to t:

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$$

$$\sec^2\left(\frac{\pi}{4}\right)(.14) = \frac{1}{500}\frac{dy}{dt}$$

d) Solve equation to find rate at which balloon is rising:

$$\frac{dy}{dt} = 500\left(\sqrt{2}\right)^2(.14)$$

→ units $\frac{ft}{min} = ft\left(\frac{rad}{min}\right)$ ✓

(radians are not a unit)

$$\frac{dy}{dt} = \boxed{140 \text{ ft/min}}$$