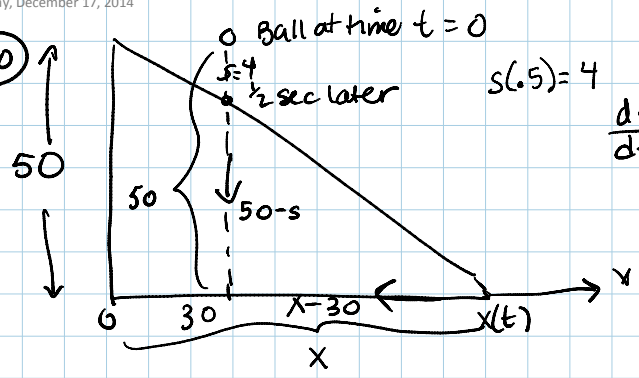
HW 4.6 p. 253 30-32, 34, 42

Wednesday, December 17, 2014
4:19 PM

(30)



Find $\frac{dx}{dt}$

$s(.5) = 4 \quad s = 16t^2$

$\frac{ds}{dt} = 32t = 32(\frac{1}{2}) = 16$

Similar Δs: $\frac{x}{50} = \frac{x-30}{50-s}$

$50x - xs = 50x - 1500$

$xs = 1500$

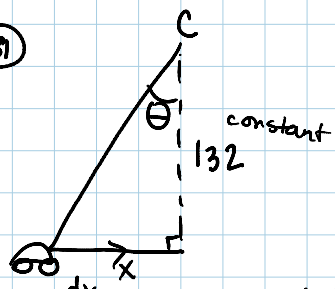
$x = \frac{1500}{s}$

$\frac{dx}{dt} = -1500s^{-2}\frac{ds}{dt}$

$\frac{dx}{dt} = -1500(4)^{-2}(16)$

$\boxed{\frac{dx}{dt} = -1500 \text{ ft/sec}}$

(31)



$\frac{dx}{dt} = 180 \text{ mph} = 264 \text{ ft/sec}$

(a) when car is right in front of you, $x = 0$, $\theta = 0$

$\tan\theta = \frac{x}{132}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{132}\frac{dx}{dt}$

$\sec^2(0) \cdot \frac{d\theta}{dt} = \frac{1}{132}(264)$

$1 \cdot \frac{d\theta}{dt} = \boxed{2 \text{ rad/sec}}$

(b) when car is $\frac{1}{2}$ sec later, $x = 264(.5) = 132$ ft/sec

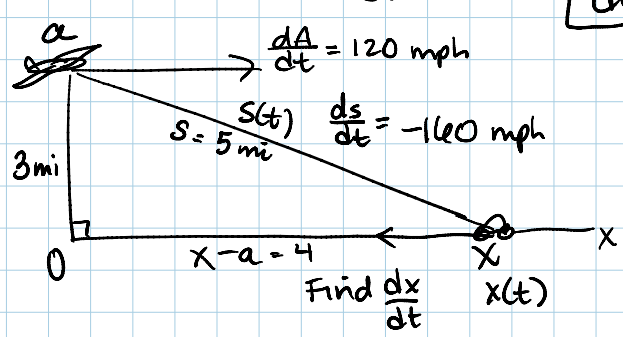
$\theta = \tan^{-1}\left(\frac{132}{132}\right) = \frac{\pi}{4}$

same deriv: $\sec^2\theta \frac{d\theta}{dt} = \frac{1}{132}\frac{dx}{dt}$

$\sec^2(\frac{\pi}{4})\frac{d\theta}{dt} = \frac{1}{132}(264)$

$(2)\frac{d\theta}{dt} = 2 \qquad \boxed{\frac{d\theta}{dt} = 1 \text{ rad/sec}}$

(32)



Find $\frac{dx}{dt}$

$3^2 + (x-a)^2 = s^2$

$0 + 2(x-a)\left(\frac{dx}{dt} - \frac{da}{dt}\right) = 2s\frac{ds}{dt}$

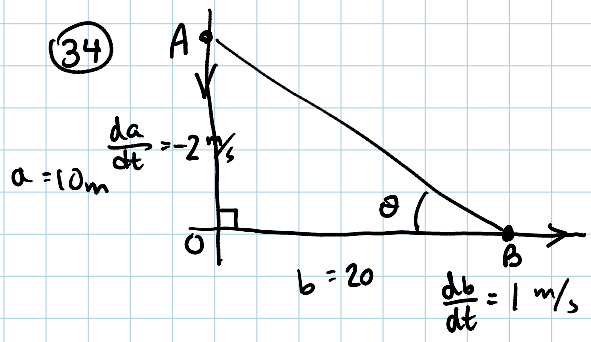
$2(4)\left(\frac{dx}{dt} - 120\right) = 2(5)(-160)$

$\frac{dx}{dt} - 120 = -200$

$$\frac{dx}{dt} = -80 \text{ mph}$$

$$\boxed{\text{Car's speed is 80 mph}}$$

(34)



$$\theta = \tan^{-1}\left(\frac{10}{20}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

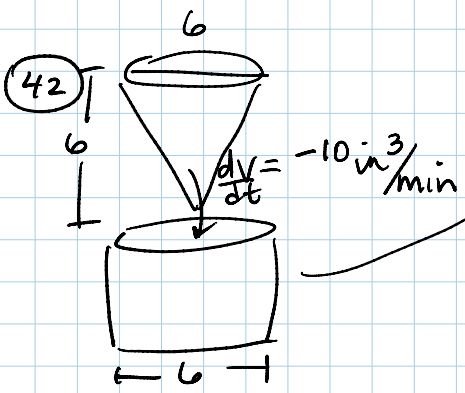
$$\tan\theta = \frac{a}{b}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{b\frac{da}{dt} - a\frac{db}{dt}}{b^2}$$

$$\sec^2\left(\tan^{-1}\left(\tfrac{1}{2}\right)\right) \cdot \frac{d\theta}{dt} = \frac{20(-2) - 10(1)}{(20)^2}$$

$$\boxed{\frac{d\theta}{dt} = -.1 \text{ rad/sec} \approx -5.730 \text{ deg/sec}}$$

(42)



(a) for cylinder, $\frac{dV}{dt} = 10$ in³/min $r = 3$ (constant)

find $\frac{dh}{dt}$

$$V_{cyl} = \pi r^2 h = 9\pi h$$

$$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$$

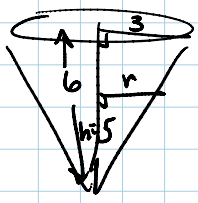
$$10 = 9\pi \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{10}{9\pi} \text{ in/min}}$$

level is rising at a rate of $\frac{10}{9\pi}$ in/min

(b) cone: $\frac{dV}{dt} = -10$ in³/min

$h = 5$



Similar Δs

$$\frac{6}{3} = \frac{h}{r}$$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi\left(3h^2\right)\frac{dh}{dt}$$

$$-10 = \frac{1}{12}\pi\left(3(5)^2\right)\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{-8}{5\pi} \text{ in/min}}$$

level is falling at a rate of $\frac{8}{5\pi}$ in/min