AP Calc - 4.5 & 4.6 Practice Problems

1a) Find the linearization $L(x)$ of $f(x)$ at x =0 if $f(x) = 3e^{2x}$

Pt: $f(0) = 3e^{2(0)} = 3$ $(0,3)$

Slope: $f'(0) = 3e^{2(0)} \cdot 2 = 6e^{2(0)} = 6$

$y - 3 = 6(x-0)$ $\quad L(x) = 3 + 6x$

1b) Use your linearization to approximate $f(0.5)$

$L(.5) = 3 + 6(.5)$

$= 6$

2) If $f(x) = 4x - x^2$, a = 1 and dx = 0.1, find

a) the true change Δf = $f(1.1) - f(1)$

$= .19$

b) the estimated change *df*

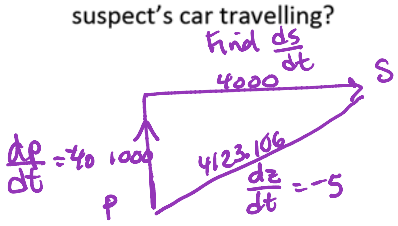
$\frac{df}{dx} = f'(x)$

$df = f'(x)dx = (4 - 2(1))(.1) = .2$

c) the approximation error

$|\Delta f - df| = .01$ $\quad$ 1% error

3) A police car is following a suspect. The suspect's car headed north to an intersection, then turned right and continued directly east. The suspect's car is 4000 meters to the east of the intersection now. The police car is 1000 meters from the intersection, headed north at 40 meters/second. If the police's radar gun measures the distance between the suspect's car and the police car to be decreasing at 5 meters/second, what speed is the suspect's car travelling?



$p^2 + s^2 = z^2$

$2p\frac{dp}{dt} + 2s\frac{ds}{dt} = 2z\frac{dz}{dt}$

$2(1000)(-40) + 2(4000)\frac{ds}{dt} = 2(4123.106)(-5)$

$\frac{ds}{dt} = 4.846$ m/s

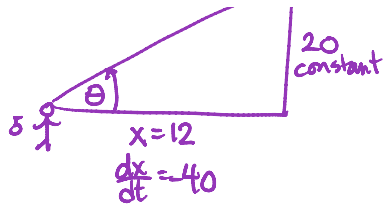
4) A model airplane is at an altitude of 25 feet above ground and a speed of 40 feet per second, on a course that will take it directly over Marcie's head. How fast will the plane's angle of elevation (from Marcie's perspective) be changing at the moment when the plane's horizontal distance from her is 12 feet? Assume that Marcie's eye level is 5 feet above ground level.

$\tan\theta = \frac{20}{12}$

$\theta = \tan^{-1}\left(\frac{20}{12}\right)$

$\tan\theta = \frac{20}{x}$

$\sec^2\theta\, d\theta = -20x^{-2}\, dx$



$\tan\theta = \frac{20}{x}$

$\sec^2\theta \frac{d\theta}{dt} = -20x^{-2}\frac{dx}{dt}$

$\sec^2(\tan^{-1}(\frac{20}{12}))\frac{d\theta}{dt} = -20(12)^{-2}(40)$

$\frac{d\theta}{dt} = 1.471 \text{ rad/sec}$