Monday, October 17, 2016

4.4 - Openers Below

4.4 - Logarithmic Differentiation Technique

****AP Problems will be Thursday instead of Wednesday****

Openers

① $y = e^{-5x}$

② $y = x^2e^x - xe^x$

③ $y = 9^{-x}$

④ $y = 3^{\cot x}$

⑤ $y = (\ln x)^2$

⑥ $y = \log_5 \sqrt{x}$

4.4 Logarithmic Differentiation

$y = x^x$ → base and exponent are both functions

$y = x^3$ Power Rule

$y = 3^x$ Exponential Rule

$\ln(y) = \ln(x^x)$ → Take Natural Log of both sides

$\ln y = x \ln x$ → power rule for logs (Product Rule!)

$\frac{1}{y}\cdot\frac{dy}{dx} = 1\cdot\ln x + x\cdot\frac{1}{x}$ → Take derivative

$\frac{1}{y}\cdot\frac{dy}{dx} = \ln x + 1$

→ Solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = y(\ln x + 1)$

$\boxed{\frac{dy}{dx} = x^x(\ln x + 1)}$ → Substitute $y = x^x$

② $y = x^{\tan x}$

$$\ln y = \ln x^{\tan x}$$

$$\ln y = \tan x \cdot \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \tan x \cdot \frac{1}{x} + \sec^2 x \cdot \ln x$$

$$\frac{dy}{dx} = y\left(\frac{\tan x}{x} + \sec^2 x \ln x\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \ln x\right)$$