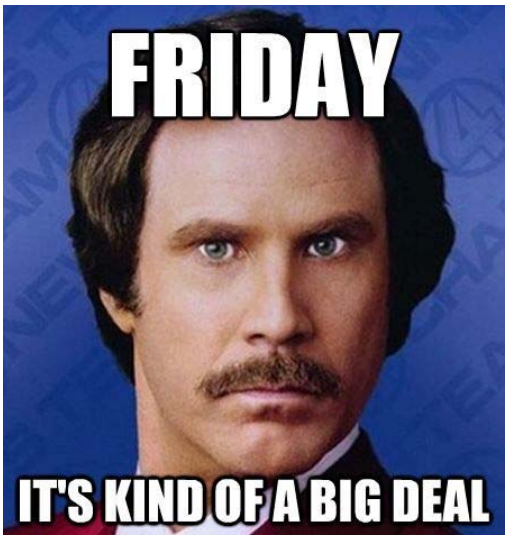
Friday, October 14, 2016

4.4 - Derivatives of Exponents and Logs

4.4 Practice Problems

NO Homework Weekend! :)

****NO Late Start on Monday!!!**



4.4 Exponential & Log Derivatives!

(1) $\frac{d}{dx} e^x = e^x$

$\left[\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}\right]$ Chain Rule

slope = graph

$y = \ln x \iff e^y = x$ change to exponential form

$e^y \cdot \frac{dy}{dx} = 1$ take derivative

$\frac{dy}{dx} = \frac{1}{e^y}$ Solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{x}$ Substitute for e^y

(2) $\frac{d}{dx} \ln x = \frac{1}{x}$

$\left[\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}\right]$ Chain Rule

$f(x)$	$f'(x)$	Power of $f'(x)$
x^3	$3x^2$	2
x^2	$2x$	1
x	1	0
1	0	0

x	1	0
$\ln x$	$\frac{1}{x}$	-1
$\frac{1}{x}$	$-x^{-2} = \frac{-1}{x^2}$	-2

Examples

(1) $y = 2e^{4x}$ find y'

$y' = 2 \cdot e^{4x} \cdot 4 = 8e^{4x}$

(2) $y = 5e^{x^2}$ find y'

$y' = 5 \cdot e^{x^2} \cdot 2x = 10xe^{x^2}$

(3) $y = \ln(3x)$ find y'

$\ln 3 + \ln x$

$0 + \frac{1}{x}$

$y' = \frac{1}{(3x)} \cdot 3 = \frac{1}{x}$

(4) $y = \log_5 x$ find y'

Change of base $y = \frac{\ln x}{\ln 5} = \frac{1}{\ln 5} \cdot \ln x$

$\frac{1}{\ln 5}$ ← constant

$y' = \frac{1}{\ln 5} \cdot \frac{1}{x} = \frac{1}{x \ln 5}$

(5) $y = \log_3 x^2$

$y = \frac{\ln x^2}{\ln 3} = \frac{1}{\ln 3} \cdot \ln x^2$

$y' = \frac{1}{\ln 3} \frac{1}{x^2} \cdot 2x = \frac{2}{x \ln 3}$

Exponentials other than $y = e^x$

x find $\frac{dy}{\quad}$ (a = constant)

$y = a^x$ find $\frac{dy}{dx}$ (a = constant)

$\ln(y) = \ln(a^x)$ take $\ln(\ )$ of both sides

$\ln y = x \ln a$ ← property of logs

($\ln a$ constant)

$\frac{1}{y} \cdot \frac{dy}{dx} = \ln a$ ← take derivative

$\frac{dy}{dx} = y \ln a$ solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = a^x \ln a$

(3) $\boxed{\frac{d}{dx} a^x = a^x \ln a}$

$\left[\frac{d}{dx} a^u = a^u \ln a \cdot \frac{du}{dx}\right]$ Chain Rule

Examples

(6) $y = 5^{x^2}$ find y'

$y' = 5^{x^2} \cdot \ln 5 \cdot 2x$

(7) $y = 3^{2x^2 + 7x}$ find y'

$y' = 3^{2x^2+7x} \cdot \ln 3 \cdot (4x + 7)$

(8) $y = \log \sqrt{x}$ ← base is 10

$y = \frac{\ln \sqrt{x}}{\ln 10} = \frac{1}{\ln 10} \cdot \ln \sqrt{x}$

$$y' = \frac{1}{\ln 10} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\ln 10} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2x \ln 10}$$