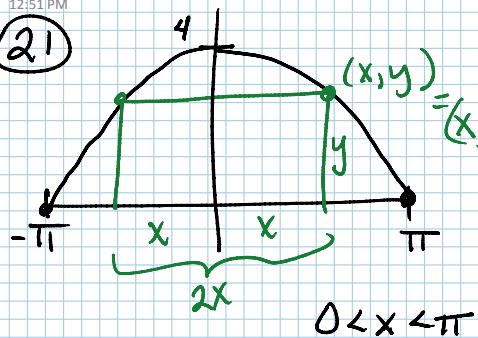


(21)



$$\frac{dA}{dx} = 8 \cos(0.5x) + 8x(-\sin(0.5x)(\frac{1}{2}))$$

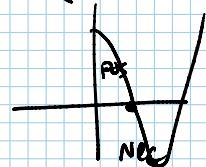
$$0 = 8 \cos(0.5x) - 4x \sin(0.5x)$$

never undefined

Use calc to find where $\frac{dA}{dx} = 0$

$x \approx 1.72$	$y \approx 2.61$
max	

$\boxed{\text{max area} = 8.98}$



(23) $r(x) = 8\sqrt{x}$ (revenue)
 $c(x) = 2x^2$ (cost)

Profit $p(x) = r(x) - c(x)$

$$p'(x) = 8(\frac{1}{2})x^{-\frac{1}{2}} - 4x$$

$$p(x) = \frac{4}{\sqrt{x}} - 4x = 0 \text{ for Max}$$

$$\frac{4}{\sqrt{x}} = 4x$$

$$4\sqrt{x} = 4$$

$$x\sqrt{x} = 1$$

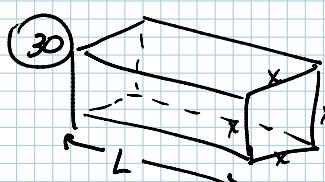
$x=1$ Max profit occurs when
 $x=1000$ units

Check $x=1$ is location of max:

$$p''(x) = 4(-\frac{1}{2})(x)^{-\frac{3}{2}} - 4$$

$$p''(x) = \frac{-2}{\sqrt{x^3}} - 4$$

$$p''(1) = \frac{-2}{\sqrt{1^3}} - 4 \text{ Neg}$$

So by 2nd deriv. test, $x=1$ is max.

Width = $4x$
Length = L

$$4x + L = 108$$

$$L = 108 - 4x$$

$$V = lwh = x(x)(L)$$

$$V = x^2(108 - 4x)$$

$$V = 108x^2 - 4x^3$$

$$\frac{dV}{dx} = 216x - 12x^2 = 0$$

Check MAX:

$$\left. \frac{d^2V}{dx^2} \right|_{x=18} = 216 - 24x \Big|_{x=18} = 216 - 432 = \text{NEG}$$

Since $V'(18)=0$ and $V''(18)<0$, local max at $x=18$.

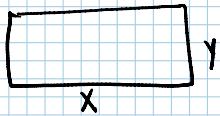
$$12x(18-x)=0$$

$$x=0, 18$$

max occurs at $x=18$ in interval $(0, 18)$

$$18 \text{ in} \times 18 \text{ in} \times 36 \text{ in}$$

(31) (a)



$$P = 2x + 2y = 36$$

$$x+y=18$$

$$y = 18 - x \quad 0 < x < 18$$



$$C = x = 2\pi r$$

$$\frac{x}{2\pi} = r$$

$$\text{Vol}_{\text{cyl}} = \pi r^2 h$$

$$= \pi \left(\frac{x}{2\pi}\right)^2 (18-x)$$

$$V = \frac{\pi x^2}{4\pi^2} (18-x)$$

$$= \frac{x^2}{4\pi} (18-x)$$

$$V = \frac{9}{2\pi} x^2 - \frac{x^3}{4\pi}$$

$$\frac{dV}{dx} = \frac{9}{\pi} x - \frac{3x^2}{4\pi} = 0$$

$$\frac{9}{\pi} x = \frac{3}{4\pi} x^2 \quad x \neq 0 \text{ so}$$

$$\frac{9}{\pi} = \frac{3}{4\pi} x$$

$$x = \frac{9}{\pi} \cdot \frac{4\pi}{3}$$

$x = 12$ check Max:

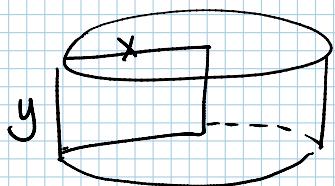
$$\left. \frac{d^2V}{dx^2} \right|_{x=12} = \frac{9}{\pi} - \frac{3x}{2\pi} \Big|_{x=12} = \text{NEG}$$

$x = 12$ is location of max
bc $V'(12) = 0$ and $V''(12) < 0$

$$\boxed{X = 12 \text{ cm}}$$

$$\boxed{Y = 6 \text{ cm}}$$

(b)



$$y = 18 - x \quad (\text{same as part (a)})$$

$$0 < x < 18$$

$$V = \pi r^2 h = \pi x^2 y = \pi x^2 (18-x) = 18\pi x^2 - \pi x^3$$

$$\frac{dV}{dx} = 36\pi x - 3\pi x^2 = 0$$

$$3\pi x(12-x) = 0$$

$$x = 0, 12 \leftarrow \text{max?}$$

$$\left. \frac{d^2V}{dx^2} \right|_{x=12} = 36\pi - 6\pi x \Big|_{x=12} = 36\pi - 6\pi(12) = \text{neg Yes!}$$

(see part (a))

$$\boxed{X = 12, Y = 6 \text{ cm}}$$

(35) (a)

$$f(x) = x^3 + ax^2 + bx$$

$$f'(x) = 3x^2 + 2ax + b$$

$$f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$$

$$3 - 2a + b = 0$$

$$+ \quad -27 - 6a - b = 0$$

$$\hline -24 - 8a = 0$$

$$8a = -24$$

$$f'(3) = 3(3)^2 + 2a(3) + b = 0$$

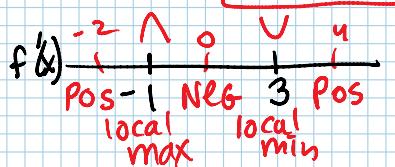
$$27 + 6a + b = 0$$

- - - - -

$$-24 - 8a = 0$$

$$8a = -24$$

$$\boxed{a = -3}$$



$$3 - 2(-3) + b = 0$$

$$\boxed{b = -9}$$

$$f(x) = 3x^2 - 6x - 9$$

(b) $f'(x) = 3x^2 + 2ax + b$

where $f'(x) = 0$

$$f'(4) = 3(4)^2 + 2a(4) + b = 0$$
$$48 + 8a + b = 0$$



POI where $f''(x) = 0$

$$f''(x) = 6x + 2a$$

$$f''(1) = 6(1) + 2a = 0$$

$$6 + 2a = 0$$

$$\boxed{a = -3}$$

$$48 + 8(-3) + b = 0$$

$$\boxed{b = -24}$$

$$f(x) = 3x^2 - 6x - 24$$