7. $\csc \frac{\pi}{4} = \sqrt{2}$

8. $\sec \frac{\pi}{3} = 2$

9. Using a right triangle with hypotenuse 13 and legs 5 (opposite) and $\sqrt{13^2 - 5^2} = 12$ (adjacent), we have $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$; $\csc\theta = \frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = \frac{12}{5}$.

10. Using a right triangle with hypotenuse 17 and legs 15 (adjacent) and $\sqrt{17^2 - 15^2} = 8$ (opposite), we have $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{8}{15}$; $\csc\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = \frac{15}{8}$.

Section 4.3 Exercises

1. The 450° angle lies on the positive y-axis ($450° - 360° = 90°$), while the others are all coterminal in Quadrant II.

2. The $-\frac{5\pi}{3}$ angle lies in Quadrant I $\left(-\frac{5\pi}{3} + 2\pi = \frac{p}{3}\right)$, while the others are all coterminal in Quadrant IV.

In #3–12, recall that the distance from the origin is $r = \sqrt{x^2 + y^2}$.

3. $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = -2$; $\csc\theta = \frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = -\frac{1}{2}$.

4. $\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = -\frac{3}{4}$; $\csc\theta = -\frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$.

5. $\sin\theta = -\frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$, $\tan\theta = 1$; $\csc\theta = -\sqrt{2}$, $\sec\theta = -\sqrt{2}$, $\cot\theta = 1$.

6. $\sin\theta = -\frac{5}{\sqrt{34}}$, $\cos\theta = \frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$; $\csc\theta = -\frac{\sqrt{34}}{5}$, $\sec\theta = \frac{\sqrt{34}}{3}$, $\cot\theta = -\frac{3}{5}$.

7. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$; $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$.

8. $\sin\theta = -\frac{3}{\sqrt{13}}$, $\cos\theta = -\frac{2}{\sqrt{13}}$, $\tan\theta = \frac{3}{2}$; $\csc\theta = -\frac{\sqrt{13}}{3}$, $\sec\theta = -\frac{\sqrt{13}}{2}$, $\cot\theta = \frac{2}{3}$.

9. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta$ undefined; $\csc\theta = 1$, $\sec\theta$ undefined, $\cot\theta = 0$.

10. $\sin\theta = 0$, $\cos\theta = -1$, $\tan\theta = 0$; $\csc\theta$ undefined, $\sec\theta = -1$, $\cot\theta$ undefined.

11. $\sin\theta = -\frac{2}{\sqrt{29}}$, $\cos\theta = \frac{5}{\sqrt{29}}$, $\tan\theta = -\frac{2}{5}$; $\csc\theta = -\frac{\sqrt{29}}{2}$, $\sec\theta = \frac{\sqrt{29}}{5}$, $\cot\theta = -\frac{5}{2}$.

12. $\sin\theta = -\frac{1}{\sqrt{2}}$, $\cos\theta = \frac{1}{\sqrt{2}}$, $\tan\theta = -1$; $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$.

For #13–16, determine the quadrant(s) of angles with the given measures, and then use the fact that $\sin t$ is positive when the terminal side of the angle is above the x-axis (in Quadrants I and II) and $\cos t$ is positive when the terminal side of the angle is to the right of the y-axis (in quadrants I and IV). Note that since $\tan t = \sin t/\cos t$, the sign of $\tan t$ can be determined from the signs of $\sin t$ and $\cos t$: If $\sin t$ and $\cos t$ have the same sign, the answer to (c) will be "+"; otherwise it will be "−". Thus $\tan t$ is positive in Quadrants I and III.

13. These angles are in Quadrant I. **(a)** + (i.e., $\sin t > 0$). **(b)** + (i.e., $\cos t > 0$). **(c)** + (i.e., $\tan t > 0$).
14. These angles are in Quadrant II. **(a)** +. **(b)** −. **(c)** −.
15. These angles are in Quadrant III. **(a)** −. **(b)** −. **(c)** +.
16. These angles are in Quadrant IV. **(a)** −. **(b)** +. **(c)** −.

For #17–20, use strategies similar to those for the previous problem set.

17. 143° is in Quadrant II, so cos 143° is negative.
18. 192° is in Quadrant III, so tan 192° is positive.
19. $\frac{7\pi}{8}$ rad is in Quadrant II, so $\cos\frac{7\pi}{8}$ is negative.
20. $\frac{4\pi}{5}$ rad is in Quadrant II, so $\tan\frac{4\pi}{5}$ is negative.
21. **(a)** $(2, 2)$; $\tan 45° = \frac{y}{x} = 1 \Rightarrow y = x$.
22. **(b)** $(-1, \sqrt{3})$; $\tan\frac{2\pi}{3} = \frac{y}{x} = -\sqrt{3}$. $\frac{2\pi}{3}$ is in Quadrant II, so x is negative.
23. **(a)** $(-\sqrt{3}, -1)$; $\frac{7\pi}{6}$ is in Quadrant III, so x and y are both negative. $\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$.
24. **(b)** $(1, -\sqrt{3})$; $-60°$ is in Quadrant IV, so x is positive while y is negative. $\tan(-60°) = -\sqrt{3}$.

For #25–36, recall that the reference angle is the acute angle formed by the terminal side of the angle in standard position and the x-axis.

25. The reference angle is 60°. A right triangle with a 60° angle at the origin has the point $P(-1, \sqrt{3})$ as one vertex, with hypotenuse length $r = 2$, so $\cos 120° = \frac{x}{r} = -\frac{1}{2}$.
26. The reference angle is 60°. A right triangle with a 60° angle at the origin has the point $P(1, -\sqrt{3})$ as one vertex, so $\tan 300° = \frac{y}{x} = -\sqrt{3}$.

27. The reference angle is the given angle, $\frac{\pi}{3}$. A right triangle with a $\frac{\pi}{3}$ radian angle at the origin has the point $P(1, \sqrt{3})$ as one vertex, with hypotenuse length $r = 2$, so $\sec\frac{\pi}{3} = \frac{r}{x} = 2$.

28. The reference angle is $\frac{\pi}{4}$. A right triangle with a $\frac{\pi}{4}$ radian angle at the origin has the point $P(1, 1)$ as one vertex, with hypotenuse length $r = \sqrt{2}$, so $\csc\frac{3\pi}{4} = \frac{r}{y} = \sqrt{2}$.

29. The reference angle is $\frac{\pi}{6}$ (in fact, the given angle is coterminal with $\frac{\pi}{6}$). A right triangle with a $\frac{\pi}{6}$ radian angle at the origin has the point $P(\sqrt{3}, 1)$ as one vertex, with hypotenuse length $r = 2$, so $\sin\frac{13\pi}{6} = \frac{y}{r} = \frac{1}{2}$.

30. The reference angle is $\frac{\pi}{3}$ (in fact, the given angle is coterminal with $\frac{\pi}{3}$). A right triangle with a $\frac{\pi}{3}$ radian angle at the origin has the point $P(1, \sqrt{3})$ as one vertex, with hypotenuse length $r = 2$, so $\cos\frac{7\pi}{3} = \frac{x}{r} = \frac{1}{2}$.

31. The reference angle is $\frac{\pi}{4}$ (in fact, the given angle is coterminal with $\frac{\pi}{4}$). A right triangle with a $\frac{\pi}{4}$ radian angle at the origin has the point $P(1, 1)$ as one vertex, so $\tan\frac{-15\pi}{4} = \frac{y}{x} = 1$.

32. The reference angle is $\frac{\pi}{4}$. A right triangle with a $\frac{\pi}{4}$ radian angle at the origin has the point $P(-1, -1)$ as one vertex, so $\cot\frac{13\pi}{4} = \frac{x}{y} = 1$.

33. $\cos\frac{23\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

34. $\cos\frac{17\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

35. $\sin\frac{11\pi}{3} = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

36. $\cot\frac{19\pi}{6} = \cot\frac{7\pi}{6} = \sqrt{3}$

37. $-450°$ is coterminal with $270°$, on the negative y-axis. **(a)** -1 **(b)** 0 **(c)** Undefined

38. $-270°$ is coterminal with $90°$, on the positive y-axis. **(a)** 1 **(b)** 0 **(c)** Undefined

39. 7π radians is coterminal with π radians, on the negative x-axis. **(a)** 0 **(b)** -1 **(c)** 0

40. $\frac{11\pi}{2}$ radians is coterminal with $\frac{3\pi}{2}$ radians, on the negative y-axis. **(a)** -1 **(b)** 0 **(c)** Undefined

41. $\frac{-7\pi}{2}$ radians is coterminal with $\frac{\pi}{2}$ radians, on the positive y-axis. **(a)** 1 **(b)** 0 **(c)** Undefined

42. -4π radians is coterminal with 0 radians, on the positive x-axis. **(a)** 0 **(b)** 1 **(c)** 0

43. Since $\cot\theta > 0$, $\sin\theta$ and $\cos\theta$ have the same sign, so $\sin\theta = +\sqrt{1-\cos^2\theta} = \frac{\sqrt{5}}{3}$, and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}}{2}$.

44. Since $\tan\theta < 0$, $\sin\theta$ and $\cos\theta$ have opposite signs, so $\cos\theta = -\sqrt{1-\sin^2\theta} = -\frac{\sqrt{15}}{4}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta} = -\sqrt{15}$.

45. $\cos\theta = +\sqrt{1-\sin^2\theta} = \frac{\sqrt{21}}{5}$, so $\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{2}{\sqrt{21}}$ and $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{\sqrt{21}}$.

46. $\sec\theta$ has the same sign as $\cos\theta$, and since $\cot\theta > 0$, $\sin\theta$ must also be negative. With $x = -3$, $y = -7$, and $r = \sqrt{3^2 + 7^2} = \sqrt{58}$, we have $\sin\theta = -\frac{7}{\sqrt{58}}$ and $\cos\theta = -\frac{3}{\sqrt{58}}$.

47. Since $\cos\theta < 0$ and $\cot\theta < 0$, $\sin\theta$ must be positive. With $x = -4$, $y = 3$, and $r = \sqrt{4^2 + 3^2} = 5$, we have $\sec\theta = -\frac{5}{4}$ and $\csc\theta = \frac{5}{3}$.

48. Since $\sin\theta > 0$ and $\tan\theta < 0$, $\cos\theta$ must be negative. With $x = -3$, $y = 4$, and $r = \sqrt{4^2 + 3^2} = 5$, we have $\csc\theta = \frac{5}{4}$ and $\cot\theta = -\frac{3}{4}$.

49. $\sin\left(\frac{\pi}{6} + 49{,}000\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

50. $\tan(1{,}234{,}567\pi) - \tan(7{,}654{,}321\pi) = \tan(\pi) - \tan(\pi) = 0$

51. $\cos\left(\frac{5{,}555{,}555\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

52. $\tan\left(\frac{3\pi - 70{,}000\pi}{2}\right) = \tan\left(\frac{3\pi}{2}\right) =$ undefined

53. The calculator's value of the irrational number π is necessarily an approximation. When multiplied by a very large number, the slight error of the original approximation is magnified sufficiently to throw the trigonometric functions off.

54. $\sin t$ is the y-coordinate of the point on the unit circle after measuring counterclockwise t units from $(1, 0)$. This will repeat every 2π units (and not before), since the distance around the circle is 2π.

55. $\mu = \frac{\sin 83°}{\sin 36°} \approx 1.69$

56. $\sin\theta_2 = \frac{\sin 42°}{1.52} \approx 0.44$

57. **(a)** When $t = 0$, $d = 0.4$ in.
(b) When $t = 3$, $d = 0.4e^{-0.6}\cos 12 \approx 0.1852$ in.