HW 4.3 p. 215 1, 3, 6, 13,15,18

Saturday, November 15, 2014 3:40 PM

① $y = x^2 - x - 1$

$y' = 2x - 1$ zero at $x = \frac{1}{2}$

$f(\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} - 1 = -\frac{5}{4}$

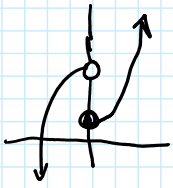
y': 0 ∪ 1 — Neg, Pos

local min at $(\frac{1}{2}, -\frac{5}{4})$ because $y' < 0$ for $x < \frac{1}{2}$ and $y' > 0$ for $x > \frac{1}{2}$

Also $(\frac{1}{2}, -\frac{5}{4})$ is absolute min b/c y is bounded below.

No max.

⑥ $y = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$



Not continuous at $x = 0$

$f(0) = 0^2 + 1 = 1$ Endpt $(0,1)$ is local min

$f'(x) = \begin{cases} -2x & \text{for } x < 0 \\ 2x & \text{for } x \geq 0 \end{cases}$ only zero at $x = 0$

③ $y = 2x^4 - 4x^2 + 1$

$y' = 8x^3 - 8x$

zeros at: $8x(x^2 - 1) = 0$

$8x(x+1)(x-1) = 0$ $x = 0, -1, 1$

y': −2 ∪ −½ ∩ ½ ∪ 2 — Neg, −1, Pos, 0, Neg, 1, Pos

local mins at $(-1, -1)$ and $(1, -1)$ bc $y' < 0$ to left of $x = -1, 1$ and $y' > 0$ to right of $x = -1, 1$

local max at $(0, 1)$ bc $y' > 0$ to left of $x = 0$ and $y' < 0$ to right of $x = 0$

local mins are also absolute

⑬ $y = xe^x$

$y' = xe^x + e^x$

$y'' = xe^x + e^x + e^x$

$y'' = e^x(x+2)$

$e^x \neq 0$, $\nexists$ und. $x + 2 = 0$ at $x = -2$

y'': −3 — Neg, −2, 0 — Pos

POI $(-2, -2e^{-2})$ bc y'' changes sign at $x = -2$

⑮ $y = \tan^{-1} x$

$y' = \frac{1}{1+x^2} \cdot 1 = (1+x^2)^{-1}$

$y'' = -(1+x^2)^{-2}(2x)$

$= \frac{-2x}{(1+x^2)^2}$

$y' = 0$ at $x = 0$

y'' never und.

y': −1, 1 — Pos, 0, Neg

POI $(0, 0)$ bc y'' changes

signs at $x=-2$

POS NEG

POI $(0,0)$ bc y'' changes signs at $x=0$

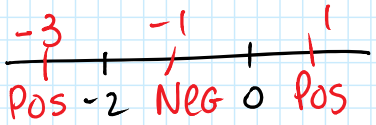
(17) $y = x^{\frac{1}{3}}(x-4)$ $\quad x^{\frac{1}{3}} \cdot x^{\frac{3}{3}} = x^{\frac{4}{3}}$

$y' = \frac{1}{3}x^{-\frac{2}{3}}(x-4) + x^{\frac{1}{3}}(1)$

$y'' = -\frac{2}{9}x^{-\frac{5}{3}}(x-4) + \frac{1}{3}x^{-\frac{2}{3}}(1) + \frac{1}{3}x^{-\frac{2}{3}}$

$= -\frac{2}{9}x^{-\frac{2}{3}} + \frac{8}{9}x^{-\frac{5}{3}} + \frac{2}{3}x^{-\frac{2}{3}}$

$= \frac{4x^{-\frac{2}{3}} + 8x^{-\frac{5}{3}}}{9} = \frac{4x+8}{x^{\frac{5}{3}}}$



POI at $(-2, 6\sqrt[3]{2})$ and $(0,0)$ bc y'' changes signs at $x=-2, 0$

y'' is und @ $x=0$

$y''=0$ at $x=-2$

(18) $y = x^{\frac{1}{2}}(x+3)$ Dom: $[0,\infty)$ *Can distribute $x^{\frac{1}{2}}$ first instead

$y' = \frac{1}{2}x^{-\frac{1}{2}}(x+3) + x^{\frac{1}{2}}(1)$

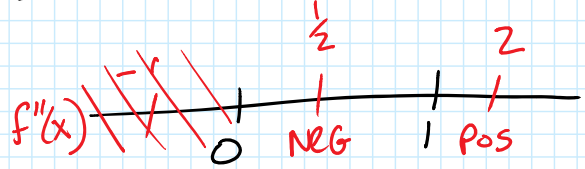
$y'' = -\frac{1}{4}x^{-\frac{3}{2}}(x+3) + \frac{1}{2}x^{-\frac{1}{2}}(1) + \frac{1}{2}x^{-\frac{1}{2}}$

$= -\frac{1}{4}x^{-\frac{1}{2}} + -\frac{3}{4}x^{-\frac{3}{2}} + x^{-\frac{1}{2}}$

$= \frac{3}{4}x^{-\frac{1}{2}} - \frac{3}{4}x^{-\frac{3}{2}}$

$= \frac{3}{4}x^{-\frac{1}{2}}(1-x^{-1})$

$y'' = \frac{3(1-\frac{1}{x})}{x^{\frac{1}{2}}}$



POI at $(1,4)$ bc y'' changes signs at $x=1$.

$y''=0$ at $x=1$

y'' und at $x=0$