16. Using a right triangle with hypotenuse 12 and legs 5 (opposite) and $\sqrt{12^2 - 5^2} = \sqrt{119}$ (adjacent), we have $\sin\theta = \frac{5}{12}$, $\cos\theta = \frac{\sqrt{119}}{12}$, $\tan\theta = \frac{5}{\sqrt{119}}$; $\csc\theta = \frac{12}{5}$, $\sec\theta = \frac{12}{\sqrt{119}}$, $\cot\theta = \frac{\sqrt{119}}{5}$.

17. Using a right triangle with hypotenuse 23 and legs 9 (opposite) and $\sqrt{23^2 - 9^2} = \sqrt{448} = 8\sqrt{7}$ (adjacent), we have $\sin\theta = \frac{9}{23}$, $\cos\theta = \frac{8\sqrt{7}}{23}$, $\tan\theta = \frac{9}{8\sqrt{7}}$; $\csc\theta = \frac{23}{9}$, $\sec\theta = \frac{23}{8\sqrt{7}}$, $\cot\theta = \frac{8\sqrt{7}}{9}$.

18. Using a right triangle with hypotenuse 17 and legs 5 (adjacent) and $\sqrt{17^2 - 5^2} = \sqrt{264} = 2\sqrt{66}$ (opposite), we have $\sin\theta = \frac{2\sqrt{66}}{17}$, $\cos\theta = \frac{5}{17}$, $\tan\theta = \frac{2\sqrt{66}}{5}$; $\csc\theta = \frac{17}{2\sqrt{66}}$, $\sec\theta = \frac{17}{5}$, $\cot\theta = \frac{5}{2\sqrt{66}}$.

19. $\frac{\sqrt{3}}{2}$

20. 1

21. $\sqrt{3}$

22. 2

23. $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

24. $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

25. $\sec 45° = 1/\cos 45° \approx 1.4142$. Squaring this result yields 2.0000, so $\sec 45° = \sqrt{2}$.

26. $\sin 60° \approx 0.8660$. Squaring this result yields $0.7500 = 3/4$, so $\sin 60° = \sqrt{3/4} = \sqrt{3}/2$.

27. $\csc(\pi/3) = 1/\sin(\pi/3) \approx 1.1547$. Squaring this result yields 1.3333 or essentially 4/3, so $\csc(\pi/3) = \sqrt{4/3} = 2/\sqrt{3} = 2\sqrt{3}/3$.

28. $\tan(\pi/3) \approx 1.73205$. Squaring this result yields 3.0000, so $\tan(\pi/3) = \sqrt{3}$.

For #29–40, the answers marked with an asterisk (*) should be found in DEGREE mode; the rest should be found in RADIAN mode. Since most calculators do not have the secant, cosecant, and cotangent functions built in, the reciprocal versions of these functions are shown.

29. ≈ 0.961*

30. ≈ 0.141*

31. ≈ 0.943*

32. ≈ 0.439*

33. ≈ 0.268

34. ≈ 0.208

35. $\frac{1}{\cos 49°} \approx 1.524$*

36. $\frac{1}{\sin 19°} \approx 3.072$*

37. $\frac{1}{\tan 0.89} \approx 0.810$

38. $\frac{1}{\cos 1.24} \approx 3.079$

39. $\frac{1}{\tan(\pi/8)} \approx 2.414$

40. $\frac{1}{\sin(\pi/10)} \approx 3.236$

41. $\theta = 30° = \frac{\pi}{6}$

42. $\theta = 60° = \frac{\pi}{3}$

43. $\theta = 60° = \frac{\pi}{3}$

44. $\theta = 45° = \frac{\pi}{4}$

45. $\theta = 60° = \frac{\pi}{3}$

46. $\theta = 45° = \frac{\pi}{4}$

47. $\theta = 30° = \frac{\pi}{6}$

48. $\theta = 30° = \frac{\pi}{6}$

49. $x = \frac{15}{\sin 34°} \approx 26.82$

50. $z = \frac{23}{\cos 39°} \approx 29.60$

51. $y = \frac{32}{\tan 57°} \approx 20.78$

52. $x = 14 \sin 43° \approx 9.55$

53. $y = \frac{6}{\sin 35°} \approx 10.46$

54. $x = 50 \cos 66° \approx 20.34$

For #55–58, choose whichever of the following formulas is appropriate:

$$a = \sqrt{c^2 - b^2} = c\sin\alpha = c\cos\beta = b\tan\alpha = \frac{b}{\tan\alpha}$$

$$b = \sqrt{c^2 - a^2} = c\cos\alpha = c\sin\beta = a\tan\beta = \frac{a}{\tan\beta}$$

$$c = \sqrt{a^2 + b^2} = \frac{a}{\cos\alpha} = \frac{a}{\sin\beta} = \frac{b}{\sin\alpha} = \frac{b}{\cos\beta}$$

If one angle is given, subtract from 90° to find the other angle.

55. $b = \frac{a}{\tan\beta} = \frac{12.3}{\tan 20°} \approx 33.79$, $c = \frac{a}{\sin\beta} = \frac{12.3}{\sin 20°} \approx 35.96$, $\alpha = 90° - \beta = 70°$

56. $a = c\sin\alpha = 10\sin 41° \approx 6.56$, $b = c\cos\alpha = 10\cos 41° \approx 7.55$, $\beta = 90° - \alpha = 49°$

57. $b = a\tan\beta = 15.58\tan 55° \approx 22.25$, $c = \frac{a}{\cos\beta} = \frac{15.58}{\cos 55°} \approx 27.16$, $\beta = 90° - \alpha = 35°$

58. $b = a\tan\beta = 5\tan 59° \approx 8.32$, $c = \frac{a}{\cos\alpha} = \frac{5}{\cos 59°} \approx 9.71$, $\alpha = 90° - \beta = 31°$

59. 0. As θ gets smaller and smaller, the side opposite θ gets smaller and smaller, so its ratio to the hypotenuse approaches 0 as a limit.

60. 1. As θ gets smaller and smaller, the side adjacent to θ approaches the hypotenuse in length, so its ratio to the hypotenuse approaches 1 as a limit.

61. $h = 55\tan 75° \approx 205.26$ ft

62. $h = 5 + 120\tan 8° \approx 21.86$ ft

63. $A = 12 \cdot \frac{5}{\sin 54°} \approx 74.16\ \text{ft}^2$

64. $h = 130\tan 82.9° \approx 1043.70$ ft

65. $AC = 100\tan 75°12'42'' \approx 378.80$ ft

66. Connect the three points on the arc to the center of the circle, forming three triangles, each with hypotenuse 10 ft. The horizontal legs of the three triangles have lengths $10\cos 67.5° \approx 3.827$, $10\cos 45° \approx 7.071$, and $10\cos 22.5° \approx 9.239$. The widths of the four strips are, therefore,

$3.827 - 0 = 3.827$ (strip A)
$7.071 - 3.827 = 3.244$ (strip B)
$9.239 - 7.071 = 2.168$ (strip C)
$10 - 9.239 = 0.761$ (strip D)

Allen needs to correct his data for strips B and C.

67. False. This is only true if θ is an acute angle in a right triangle. (Then it is true by definition.)

68. False. The larger the angle of a triangle, the smaller its cosine.

Quick Review 4.2

1. $x = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$
2. $x = \sqrt{8^2 + 12^2} = \sqrt{208} = 4\sqrt{13}$
3. $x = \sqrt{10^2 - 8^2} = 6$
4. $x = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$
5. $8.4 \text{ ft} \cdot 12 \frac{\text{in.}}{\text{ft}} = 100.8 \text{ in.}$
6. $940 \text{ ft} \cdot \frac{1}{5280} \frac{\text{mi}}{\text{ft}} = \frac{47}{264} \approx 0.17803 \text{ mi}$
7. $a = (0.388)(20.4) = 7.9152 \text{ km}$
8. $b = \frac{23.9}{1.72} \approx 13.895 \text{ ft}$
9. $\alpha = 13.3 \cdot \frac{2.4}{31.6} \approx 1.0101$ (no units)
10. $\beta = 5.9 \cdot \frac{6.15}{8.66} \approx 4.18995$ (no units)

Section 4.2 Exercises

1. $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = \frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = \frac{3}{4}.$
2. $\sin\theta = \frac{8}{\sqrt{113}}, \cos\theta = \frac{7}{\sqrt{113}}, \tan\theta = \frac{8}{7}; \csc\theta = \frac{\sqrt{113}}{8}, \sec\theta = \frac{\sqrt{113}}{7}, \cot\theta = \frac{7}{8}.$
3. $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}; \csc\theta = \frac{13}{12}, \sec\theta = \frac{13}{5}, \cot\theta = \frac{5}{12}.$
4. $\sin\theta = \frac{8}{17}, \cos\theta = \frac{15}{17}, \tan\theta = \frac{8}{15}; \csc\theta = \frac{17}{8}, \sec\theta = \frac{17}{15}, \cot\theta = \frac{15}{8}.$
5. The hypotenuse length is $\sqrt{7^2 + 11^2} = \sqrt{170}$, so $\sin\theta = \frac{7}{\sqrt{170}}, \cos\theta = \frac{11}{\sqrt{170}}, \tan\theta = \frac{7}{11}; \csc\theta = \frac{\sqrt{170}}{7}, \sec\theta = \frac{\sqrt{170}}{11}, \cot\theta = \frac{11}{7}.$
6. The adjacent side length is $\sqrt{8^2 - 6^2} = \sqrt{28} = 2\sqrt{7}$, so $\sin\theta = \frac{3}{4}, \cos\theta = \frac{\sqrt{7}}{4}, \tan\theta = \frac{3}{\sqrt{7}}; \csc\theta = \frac{4}{3}, \sec\theta = \frac{4}{\sqrt{7}}, \cot\theta = \frac{\sqrt{7}}{3}.$
7. The opposite side length is $\sqrt{11^2 - 8^2} = \sqrt{57}$, so $\sin\theta = \frac{\sqrt{57}}{11}, \cos\theta = \frac{8}{11}, \tan\theta = \frac{\sqrt{57}}{8}; \csc\theta = \frac{11}{\sqrt{57}}, \sec\theta = \frac{11}{8}, \cot\theta = \frac{8}{\sqrt{57}}.$
8. The adjacent side length is $\sqrt{13^2 - 9^2} = \sqrt{88} = 2\sqrt{22}$, so $\sin\theta = \frac{9}{13}, \cos\theta = \frac{2\sqrt{22}}{13}, \tan\theta = \frac{9}{2\sqrt{22}}; \csc\theta = \frac{13}{9}, \sec\theta = \frac{13}{2\sqrt{22}}, \cot\theta = \frac{2\sqrt{22}}{9}.$
9. Using a right triangle with hypotenuse 7 and legs 3 (opposite) and $\sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$ (adjacent), we have $\sin\theta = \frac{3}{7}, \cos\theta = \frac{2\sqrt{10}}{7}, \tan\theta = \frac{3}{2\sqrt{10}}; \csc\theta = \frac{7}{3}, \sec\theta = \frac{7}{2\sqrt{10}}, \cot\theta = \frac{2\sqrt{10}}{3}.$
10. Using a right triangle with hypotenuse 3 and legs 2 (opposite) and $\sqrt{3^2 - 2^2} = \sqrt{5}$ (adjacent), we have $\sin\theta = \frac{2}{3}, \cos\theta = \frac{\sqrt{5}}{3}, \tan\theta = \frac{2}{\sqrt{5}}; \csc\theta = \frac{3}{2}, \sec\theta = \frac{3}{\sqrt{5}}, \cot\theta = \frac{\sqrt{5}}{2}.$
11. Using a right triangle with hypotenuse 11 and legs 5 (adjacent) and $\sqrt{11^2 - 5^2} = \sqrt{96} = 4\sqrt{6}$ (opposite), we have $\sin\theta = \frac{4\sqrt{6}}{11}, \cos\theta = \frac{5}{11}, \tan\theta = \frac{4\sqrt{6}}{5}; \csc\theta = \frac{11}{4\sqrt{6}}, \sec\theta = \frac{11}{5}, \cot\theta = \frac{5}{4\sqrt{6}}.$
12. Using a right triangle with hypotenuse 8 and legs 5 (adjacent) and $\sqrt{8^2 - 5^2} = \sqrt{39}$ (opposite), we have $\sin\theta = \frac{\sqrt{39}}{8}, \cos\theta = \frac{5}{8}, \tan\theta = \frac{\sqrt{39}}{5}; \csc\theta = \frac{8}{\sqrt{39}}, \sec\theta = \frac{8}{5}, \cot\theta = \frac{5}{\sqrt{39}}.$
13. Using a right triangle with legs 5 (opposite) and 9 (adjacent) and hypotenuse $\sqrt{5^2 + 9^2} = \sqrt{106}$, we have $\sin\theta = \frac{5}{\sqrt{106}}, \cos\theta = \frac{9}{\sqrt{106}}, \tan\theta = \frac{5}{9}; \csc\theta = \frac{\sqrt{106}}{5}, \sec\theta = \frac{\sqrt{106}}{9}, \cot\theta = \frac{9}{5}.$
14. Using a right triangle with legs 12 (opposite) and 13 (adjacent) and hypotenuse $\sqrt{12^2 + 13^2} = \sqrt{313}$, we have $\sin\theta = \frac{12}{\sqrt{313}}, \cos\theta = \frac{13}{\sqrt{313}}, \tan\theta = \frac{12}{13}; \csc\theta = \frac{\sqrt{313}}{12}, \sec\theta = \frac{\sqrt{313}}{13}, \cot\theta = \frac{13}{12}.$
15. Using a right triangle with legs 3 (opposite) and 11 (adjacent) and hypotenuse $\sqrt{3^2 + 11^2} = \sqrt{130}$, we have $\sin\theta = \frac{3}{\sqrt{130}}, \cos\theta = \frac{11}{\sqrt{130}}, \tan\theta = \frac{3}{11}; \csc\theta = \frac{\sqrt{130}}{3}, \sec\theta = \frac{\sqrt{130}}{11}, \cot\theta = \frac{11}{3}.$