

3.7 Implicit Differentiation

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12:25 PM

① Graph $x^2 + y^2 = 16$

and find slope of tangent
to the curve at $(3, -\sqrt{7})$

Need derivative at $(3, -\sqrt{7})$

OLD WAY: $\sqrt{y^2} = \sqrt{16 - x^2}$
 $y = \pm \sqrt{16 - x^2}$

for $(3, -\sqrt{7})$: $y = -\sqrt{16 - x^2} = -(16 - x^2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = -\frac{1}{2}(16 - x^2)^{-\frac{1}{2}}(-2x) = x(16 - x^2)^{-\frac{1}{2}} = \frac{x}{\sqrt{16 - x^2}}$$

$$\left. \frac{dy}{dx} \right|_{x=3} = \frac{3}{\sqrt{16 - 3^2}} = \frac{3}{\sqrt{7}}$$

NEW WAY: Implicit Diff:

$$x^2 + y^2 = 16$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} 16$$

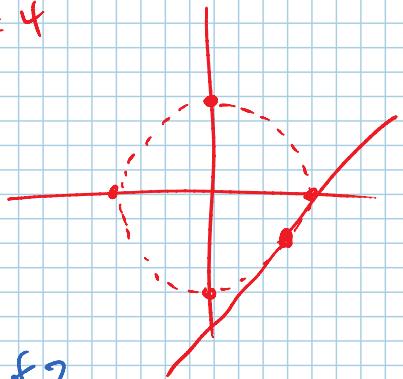
$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \cdot \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\left. \frac{dy}{dx} \right|_{(3, -\sqrt{7})} = \frac{-3}{-\sqrt{7}} = \frac{3}{\sqrt{7}}$$

circle
center $(0, 0)$
 $r = 4$



circle
made of 2
functions

* Take der. w/ respect to x
on both sides

* For terms w/ y, use
Chain Rule

* Solve for $\frac{dy}{dx}$

$$\textcircled{2} \quad 2y = x^2 + \sin y \quad \text{Find deriv. } \left(\frac{dy}{dx} \right)$$

$$\frac{d}{dx}(2y) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin y)$$

$$\underbrace{2 \cdot \frac{dy}{dx}}_{2} = 2x + \cos y \cdot \underbrace{\frac{dy}{dx}}_{\text{cosy}}$$

$$2 \frac{dy}{dx} - \cos y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2 - \cos y) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

$$\textcircled{3} \quad x^2 + xy - y^2 = 1 \quad \text{Find } \frac{dy}{dx}.$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy - \frac{d}{dx}y^2 = \frac{d}{dx}1$$

$$2x + \underbrace{1 \cdot y + x \cdot 1 \cdot \frac{dy}{dx}}_{1 \cdot y + x \cdot \frac{dy}{dx}} - 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x - 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y}$$