HW 3.7 Day 1 p. 162 #1, 2, 5, 18, 19, 21

Thursday, July 3, 2014
11:49 AM

① $x^2y + xy^2 = 6$

$\frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) = \frac{d}{dx}(6)$

$2xy + x^2\frac{dy}{dx} + 1\cdot y^2 + 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2+2xy) = -2xy - y^2$

$\boxed{\frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}}$

② $x^3 + y^3 = 18xy$

$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}18xy$

$3x^2 + 3y^2\frac{dy}{dx} = 18(1\cdot y + x\cdot\frac{dy}{dx})$

$3x^2 + 3y^2\frac{dy}{dx} = 18y + 18x\frac{dy}{dx}$

$\frac{dy}{dx}(3y^2 - 18x) = 18y - 3x^2$

$\frac{dy}{dx} = \frac{18y-3x^2}{3y^2-18x} = \boxed{\frac{6y-x^2}{y^2-6x}}$

$\frac{\cancel{3}(6y-x^2)}{\cancel{3}(y^2-6x)}$

⑤ $x = \tan y$

$\frac{d}{dx}x = \frac{d}{dx}\tan y$

$1 = \sec^2 y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \boxed{\cos^2 y}$

18. $x^2 + y^2 = 25 \quad (3,-4)$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\left.\frac{dy}{dx}\right|_{(3,-4)} = \frac{3}{4}$ = slope of tangent

ⓐ tan line:

$y+4 = \frac{3}{4}(x-3)$

$\boxed{y = \frac{3}{4}x - \frac{25}{4}}$ $\quad -\frac{9}{4} - \frac{16}{4}$

ⓑ normal line $m = -\frac{4}{3}$

$y+4 = -\frac{4}{3}(x-3)$

$\boxed{y = -\frac{4}{3}x}$

19. $x^2y^2 = 9 \quad (-1,3)$

$2xy^2 + 2y\frac{dy}{dx}x^2 = 0$

$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = \frac{-y}{x}$

$\left.\frac{dy}{dx}\right|_{(-1,3)} = \frac{-3}{-1} = 3$ = slope of tangent

ⓐ tan line:

$y-3 = 3(x+1)$

$\boxed{y = 3x+6}$

ⓑ normal line: $m = -\frac{1}{3}$

$y-3 = -\frac{1}{3}(x+1)$

$\boxed{y = -\frac{1}{3}x + \frac{8}{3}}$

21. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0 \quad (-1,0)$

$12x + 3(1y + 1\frac{dy}{dx}x) + 4y\cdot\frac{dy}{dx} + 17\frac{dy}{dx} + 0 = 0$

$12x + 3y + \frac{dy}{dx}3x + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-12x-3y}{3x+4y+17}$

$$\left.\frac{dy}{dx}\right|_{(-1,0)} = \frac{-12(-1)-3(0)}{3(-1)+0+17} = \frac{12}{14} = \frac{6}{7}$$

ⓐ tangent: $y = \frac{6}{7}(x+1)$

$$\boxed{y = \frac{6}{7}x + \frac{6}{7}}$$

ⓑ normal $y = \frac{-7}{6}(x+1)$

$$\boxed{y = -\frac{7}{6}x - \frac{7}{6}}$$