AP Calculus AB
Worksheet 3.7-3.8 Name____________________

1. Find y' of $x^3 + y^3 = 18xy$.

$$\frac{d}{dx}x^3 + \frac{d}{dx}y^3 = \frac{d}{dx}18xy$$

$$3x^2 + 3y^2\frac{dy}{dx} = 18\left(y + x\frac{dy}{dx}\right)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 18y + 18x\frac{dy}{dx}$$

$$\frac{dy}{dx}(3y^2 - 18x) = 18y - 3x^2$$

$$\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x} = \boxed{\frac{6y - x^2}{y^2 - 6x}}$$

2. Find y' and y'' of $y^2 = 1 - 2x^2$. Simplify completely.

$$y^2 + 2x^2 = 1$$

$$2y\frac{dy}{dx} = 0 - 4x$$

$$\frac{dy}{dx} = \frac{-4x}{2y} = \boxed{\frac{-2x}{y}} = y'$$

$$\frac{d^2y}{dx^2} = \frac{y(-2) - (-2x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-2y + 2x\left(\frac{-2x}{y}\right)}{y^2}$$

$$= \frac{-2y - \frac{4x^2}{y}}{y^2} \cdot \frac{y}{y} =$$

$$= \frac{-2y^2 - 4x^2}{y^3} = \frac{-2(y^2 + 2x^2)}{y^3}$$

$$= \frac{-2(1)}{y^3} = \boxed{\frac{-2}{y^3}}$$

3. Find $\frac{d^2y}{dx^2}$ of $x^2 + y^2 = 6$. Simplify completely.

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \boxed{\frac{-x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$= \frac{-y - \frac{x^2}{y}}{y^2} \cdot \frac{y}{y}$$

$$= \frac{-y^2 - x^2}{y^3} = \frac{-(y^2 + x^2)}{y^3} = \boxed{\frac{-6}{y^3}}$$

4. Find dy/dx of each:

a) $y = \sin^{-1}(4x^2+3)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x^2+3)^2}} \cdot 8x$$

$$= \frac{8x}{\sqrt{1-16x^2-24x-9}}$$

$$= \frac{8x}{\sqrt{-16x^2-24x-8}} = \frac{8x}{\sqrt{4(-4x^2-6x-2)}}$$

$$= \frac{\cancel{8}x}{\cancel{2}\sqrt{-4x^2-6x-2}} = \boxed{\frac{4x}{\sqrt{-4x^2-6x-2}}}$$

b) $y = 4x + \cot^{-1}(16\sqrt{x})$

$16x^{1/2}$

$$\frac{dy}{dx} = 4 + \frac{-1}{1+(16\sqrt{x})^2} \cdot 8x^{-1/2}$$

$$= 4 + \frac{-8}{\sqrt{x}\,(1+256x)}$$

$$= \boxed{4 - \frac{8}{\sqrt{x}+256x^{3/2}}}$$

5. Find the equations of the tangent and normal lines of $y = \sec^{-1}(2x)$ at x = 1.

$$m = \frac{dy}{dx} = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 = \frac{2}{2|x|\sqrt{4x^2-1}} = \frac{1}{|x|\sqrt{4x^2-1}}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1\sqrt{4-1}} = \frac{1}{\sqrt{3}}$$

point $_{x=1}$ $y = \sec^{-1}(2\cdot 1) = \frac{\pi}{3}$

$(1, \frac{\pi}{3})$

$$\boxed{\begin{array}{l} \text{Tangent: } y - \frac{\pi}{3} = \frac{1}{\sqrt{3}}(x-1) \\ \text{Normal: } y - \frac{\pi}{3} = -\sqrt{3}(x-1) \end{array}}$$