3.6 Notes Day 1

Monday, November 21, 2016 8:55 AM

The Mathematics of Finance

Section 3.6

INTEREST COMPOUNDED ANNUALLY

The value of an investment after n years is represented by the following formula:

$$A = P(1+r)^t$$

A = amount after t # of years
P = principal (money you started with)
r = rate (decimal)
t = time in years

1) Find the value of a $10,000 investment after 5 years at a rate of 2.5%, compounded annually.

$A = 10{,}000(1+.025)^5 = \$11{,}314.08$ (.025)

INTEREST COMPOUNDED k TIMES PER YEAR

$$A = P\left(1+\frac{r}{k}\right)^{kt}$$

k = # of times compounded per year
monthly = 12 = k
quarterly = 4 = k
daily = 365 = k
t = # of years

2) Using the info from Problem #1, find the value of the same investment if it is compounded

a) monthly (k=12) $A = 10000\left(1+\frac{.025}{12}\right)^{12(5)} = \$11{,}330.01$

b) daily (k=365) $A = 10{,}000\left(1+\frac{.025}{365}\right)^{365(5)} = \$11{,}331.43$

3) Joe invests $500 at 4% annual interest rate compounded quarterly. Find the value of Joe's investment in 10 years.

$744.43

FINDING THE TIME PERIOD OF AN INVESTMENT

k=12

4) Millie has $1200 to invest at 5.5% annual interest rate compounded monthly. She desperately wants to buy a used car she saw advertised for $3800. Assuming the car is still available and the offer is still good, when would she be able to purchase the car?

$$\frac{3800}{1200} = \frac{1200}{1200}\left(1+\frac{.055}{12}\right)^{12t}$$

$$\frac{38}{12} = \left(1+\frac{.055}{12}\right)^{12t}$$

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$$\frac{38}{12} = \left(1 + \frac{.055}{12}\right)^{12t}$$

$$12t = \log_{\left(1+\frac{.055}{12}\right)} \frac{38}{12}$$

$$t \simeq 21 \text{ years}$$

5) How long would it take you to double your money on a $1000 investment at an annual interest rate of 10%, compounded annually?

$2000 = 1000(1 + .1)^t$

$t \approx 7.27$ years

Doubling Time for Money = 72 ÷ Interest Rate

FINDING THE INTEREST RATE ON AN INVESTMENT

6) Koko has $2500 to invest. What annual interest rate compounded quarterly is required for him to double his money in 10 years?

7) If you invest $100,000 now, and want to have $1,000,000 when you retire in 30 years, what annual interest rate is required if it is compounded daily?

INTEREST COMPOUNDED CONTINUOUSLY

The value of an investment compounded continuously (versus a certain number of times per year) is represented by the following formula:

$A = Pe^{rt}$

A = amt after t years
P = principal
e = natural base ≈ 2.718...
r = rate
t = # of years

8) Use the information from Problem #1 to compute the value of the investment if interest is compounded continuously. P = 10,000 r = 2.5%

$A = 10{,}000e^{.025(5)} = \$11{,}331.48$

9) If you invest $1000 at a 4.5% annual interest rate, compounded continuously, how long will it take for your money to double?