Section 3.6 Exercises

1. Compound interest: $A = 1500(1 + 0.07)^6 \approx \$2251.10$; simple interest: $A = 1500(1 + 0.07(6)) \approx \$2130$.
2. Compound interest: $A = 3200(1 + 0.08)^4 \approx \$4353.56$; simple interest: $A = 3200(1 + 0.08(4)) \approx \$4224$.
3. Compound interest: $A = 12{,}000(1 + 0.075)^7 \approx \$19{,}908.59$; simple interest: $A = 12{,}000(1 + 0.075(7)) \approx \$18{,}300$.
4. Compound interest: $A = 15{,}500(1 + 0.095)^{12} \approx \$46{,}057.58$; simple interest: $A = 15{,}500(1 + 0.095(12)) \approx \$33{,}170$.
5. $A = 1500\left(1 + \frac{0.07}{4}\right)^{20} \approx \$2122.17$
6. $A = 3500\left(1 + \frac{0.05}{4}\right)^{40} \approx \$5752.67$
7. $A = 40{,}500\left(1 + \frac{0.038}{12}\right)^{240} \approx \$86{,}496.26$
8. $A = 25{,}300\left(1 + \frac{0.045}{12}\right)^{300} \approx \$77{,}765.69$
9. $A = 1250e^{(0.054)(6)} \approx \$1728.31$
10. $A = 3350e^{(0.062)(8)} \approx \$5501.17$
11. $A = 21{,}000e^{(0.037)(10)} \approx \$30{,}402.43$
12. $A = 8875e^{(0.044)(25)} \approx \$26{,}661.97$
13. $FV = 500 \cdot \dfrac{\left(1 + \frac{0.07}{4}\right)^{24} - 1}{\frac{0.07}{4}} \approx \$14{,}755.51$
14. $FV = 300 \cdot \dfrac{\left(1 + \frac{0.06}{4}\right)^{48} - 1}{\frac{0.06}{4}} \approx \$20{,}869.57$
15. $FV = 450 \cdot \dfrac{\left(1 + \frac{0.0525}{12}\right)^{120} - 1}{\frac{0.0525}{12}} \approx \$70{,}819.63$
16. $FV = 610 \cdot \dfrac{\left(1 + \frac{0.065}{12}\right)^{300} - 1}{\frac{0.065}{12}} \approx \$456{,}790.28$
17. $PV = 815.37 \cdot \dfrac{1 - \left(1 + \frac{0.047}{12}\right)^{-60}\frac{0.047}{12}}{} \approx \$43{,}523.31$
18. $PV = 1856.82 \cdot \dfrac{1 - \left(1 + \frac{0.065}{12}\right)^{-360}\frac{0.065}{12}}{} \approx \$293{,}769.01$

19. $R = \frac{PV \cdot i}{1 - (1+i)^{-n}} = \frac{(18{,}000)\left(\frac{0.054}{12}\right)}{1 - \left(1 + \frac{0.054}{12}\right)^{-72}} \approx \$293.24$

20. $R = \frac{PV \cdot i}{1 - (1+i)^{-n}} = \frac{(154{,}000)\left(\frac{0.072}{12}\right)}{1 - \left(1 + \frac{0.072}{12}\right)^{-180}} \approx \$1401.47$

In #21–24, the time must be rounded up to the end of the next compounding period.

21. Solve $2300\left(1 + \frac{0.09}{4}\right)^{4t} = 4150{:}(1.0225)^{4t} = \frac{83}{46}$, so $t = \frac{1}{4}\frac{\ln(83/46)}{\ln 1.0225} \approx 6.63$ years — round to 6 years 9 months (the next full compounding period).

22. Solve $8000\left(1 + \frac{0.09}{12}\right)^{12t} = 16{,}000{:}(1.0075)^{12t} = 2$, so $t = \frac{1}{12}\frac{\ln 2}{\ln 1.0075} \approx 7.73$ years — round to 7 years 9 months (the next full compounding period).

23. Solve $15{,}000\left(1 + \frac{0.08}{12}\right)^{12t} = 45{,}000{:}(1.0067)^{12t} = 3$, so $t = \frac{1}{12}\frac{\ln 3}{\ln 1.0067} \approx 13.71$ years — round to 13 years 9 months (the next full compounding period). Note: A graphical solution provides $t \approx 13.78$ years — round to 13 years 10 months.

24. Solve $1.5\left(1 + \frac{0.08}{4}\right)^{4t} = 3.75{:}(1.02)^{4t} = 2.5$, so $t = \frac{1}{4}\frac{\ln 2.5}{\ln 1.02} \approx 11.57$ years — round to 11 years 9 months (the next full compounding period).

25. Solve $22{,}000\left(1 + \frac{r}{365}\right)^{(365)(5)} = 36{,}500$:
$1 + \frac{r}{365} = \left(\frac{73}{44}\right)^{1/1825}$, so $r \approx 10.13\%$.

26. Solve $8500\left(1 + \frac{r}{12}\right)^{(12)(5)} = 3 \cdot 8500$:
$1 + \frac{r}{12} = 3^{1/60}$, so $r \approx 22.17\%$.

27. Solve $14.6\,(1 + r)^6 = 22$: $1 + r = \left(\frac{110}{73}\right)^{1/6}$, so $r \approx 7.07\%$.

28. Solve $18\,(1 + r)8 = 25$: $1 + r = \left(\frac{25}{18}\right)^{1/8}$, so $r \approx 4.19\%$.

In #29 and 30, the time must be rounded up to the end of the next compounding period.

29. Solve $\left(1 + \frac{0.0575}{4}\right)^{4t} = 2$: $t = \frac{1}{4}\frac{\ln 2}{\ln 1.014375} \approx 12.14$ — round to 12 years 3 months.

30. Solve $\left(1 + \frac{0.0625}{12}\right)^{12t} = 3$:
$t = \frac{1}{12}\frac{\ln 3}{\ln(1 + 0.0625/12)} \approx 17.62$ — round to 17 years 8 months.

For #31–34, use the formula $S = Pe^{rt}$.

31. Time to double: Solve $2 = e^{0.09t}$, leading to $t = \frac{1}{0.09}\ln 2 \approx 7.7016$ years. After 15 years: $S = 12{,}500e^{(0.09)(15)} \approx \$48{,}217.82$.

32. Time to double: Solve $2 = e^{0.08t}$, leading to $t = \frac{1}{0.08}\ln 2 \approx 8.6643$ years. After 15 years: $S = 32{,}500e^{(0.08)(15)} \approx \$107{,}903.80$.

33. APR: Solve $2 = e^{4r}$, leading to $r = \frac{1}{4}\ln 2 \approx 17.33\%$.
After 15 years: $S = 9500e^{(0.1733)(15)} \approx \$127{,}816.26$ (using the "exact" value of r).

34. APR: Solve $2 = e^{6r}$, leading to $r = \frac{1}{6}\ln 2 \approx 11.55\%$.
After 15 years: $S = 16{,}800e^{(0.1155)(15)} \approx \$95{,}035.15$ (using the "exact" value of r).

In #35–41, the time must be rounded up to the end of the next compounding period (except in the case of continuous compounding).

35. Solve $\left(1 + \frac{0.04}{4}\right)^{4t} = 2$: $t = \frac{1}{4}\frac{\ln 2}{\ln 1.01} \approx 17.42$, which rounds to 17 years 6 months.

36. Solve $\left(1 + \frac{0.08}{4}\right)^{4t} = 2$: $t = \frac{1}{4}\frac{\ln 2}{\ln 1.02} \approx 8.751$, which rounds to 9 years (*almost* by 8 years 9 months).

37. Solve $1 + 0.07t = 2$: $t = \frac{1}{0.07} \approx 14.29$, which rounds to 15 years.

38. Solve $1.07^t = 2$: $t = \frac{\ln 2}{\ln 1.07} \approx 10.24$, which rounds to 11 years.

39. Solve $\left(1 + \frac{0.07}{4}\right)^{4t} = 2$: $t = \frac{1}{4}\frac{\ln 2}{\ln 1.0175} \approx 9.99$, which rounds to 10 years.

40. Solve $\left(1 + \frac{0.07}{12}\right)^{12t} = 2$: $t = \frac{1}{12}\frac{\ln 2}{\ln(1 + 0.07/12)}$ ≈ 9.93, which rounds to 10 years.

41. Solve $e^{0.07t} = 2$: $t = \frac{1}{0.07}\ln 2 \approx 9.90$ years.

For #42–45, observe that the initial balance has no effect on the APY.

42. $\text{APY} = \left(1 + \frac{0.06}{4}\right)^4 - 1 \approx 6.14\%$

43. $\text{APY} = \left(1 + \frac{0.0575}{365}\right)^{365} - 1 \approx 5.92\%$

44. $\text{APY} = e^{0.063} - 1 \approx 6.50\%$

45. $\text{APY} = \left(1 + \frac{0.047}{12}\right)^{12} - 1 \approx 4.80\%$

46. The APYs are $\left(1 + \frac{0.05}{12}\right)^{12} - 1 \approx 5.1162\%$ and $\left(1 + \frac{0.051}{4}\right)^4 - 1 \approx 5.1984\%$. So, the better investment is 5.1% compounded quarterly.

47. The APYs are $5\frac{1}{8}\% = 5.125\%$ and $e^{0.05} - 1 \approx 5.1271\%$.

So, the better investment is 5% compounded continuously.

For #48–51, use the formula $S = R\frac{(1+i)^n - 1}{i}$.

48. $i = \frac{0.0726}{12} = 0.00605$ and $R = 50$, so

$$S = 50\,\frac{(1.00605)^{(12)(25)} - 1}{0.00605} \approx \$42{,}211.46.$$

49. $i = \frac{0.155}{12} = 0.0129\ldots$ and $R = 50$, so

$$S = 50\,\frac{(1.0129)^{(12)(20)} - 1}{0.0129} \approx \$80{,}367.73.$$

50. $i = \frac{0.124}{12}$; solve

$$250{,}000 = R\,\frac{\left(1 + \frac{0.124}{12}\right)^{(12\times 30)} - - 1}{\frac{0.124}{12}}$$ to obtain

$R \approx \$239.42$ per month (round up, since \$239.41 will not be adequate).

51. $i = \frac{0.045}{12} = 0.00375$; solve

$$120{,}000 = R\,\frac{(1.00375)^{(12)(30)} - 1}{0.00375}$$ to obtain $R \approx \$158.03$

per month (round up, since \$158.02 will not be adequate).

For #52–55, use the formula $A = R\frac{1 - (1+i)^{-n}}{i}$.

52. $i = \frac{0.0795}{12} = 0.006625$; solve

$$9000 = R\,\frac{1 - (1.006625)^{-(12)(4)}}{0.006625}$$ to obtain $R \approx \$219.51$

per month.