Section 3.6 Name _____
Annuities Problems
Future Value/Present Value

*An **annuity** is a sequence of equal periodic payments. The annuity of ordinary deposits are made at the end of each period at the same time the interest is posted in the account.*

LOANS AND MORTGAGES – PRESENT VALUE

An annuity is a sequence of periodic payments. The net amount of money put into an annuity is its **PRESENT VALUE.** The net amount returned from the annuity is its **FUTURE VALUE.**

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

PV = present value (amt of loan)

R = monthly payment

**i = interest rate per period $\frac{APR}{K} = \frac{\text{rate}}{K}$

n = number of payments = Kt

K = # times compounded per year

t = time in yrs

APR = annual percentage rate

1) Chucky purchases a new pick-up truck for $18,500. What are the monthly payments for a 4-year loan with $2000 down payment if the annual interest rate (APR) is 2.9%.

PV = 18,500 − 2000 R? t = 4

K = 12 $i = \frac{.029}{12}$ $n = 12(4) = 48$

$$16{,}500 = R\frac{1-\left(1+\frac{.029}{12}\right)^{-48}}{\frac{.029}{12}}$$

$16{,}500 = R(45.27)$

R = $364.49

How much interest does he pay over life of loan?

Rn = $17,495.52
− 16 500
$995.52

2) Gilda obtains a 30-year mortgage of $286,000 with an APR of 4.75%. What is her monthly payment?

PV t = 12

$$286{,}000 = R\frac{1-\left(1+\frac{.0475}{12}\right)^{-360}}{\frac{.0475}{12}}$$

$i = \frac{.0475}{12}$

$n = 30(12) = 360$

$286{,}000 = R(191.70)$

R = $1491.91

How much interest?

Rn = $537,088
− 286,000
$251,088 !

So, the total value of the investment returned from the annuity consists of all the periodic payments together with all the interest. This value is called the **FUTURE VALUE** of the annuity because it is typically calculated when projecting into the future ☺.

FUTURE VALUE OF AN ANNUITY:

$$FV = R\frac{(1+i)^n - 1}{i}$$

FV = future value

R = monthly payments

****i** = interest rate per period $i = \frac{APR}{k}$

n = number of payments $n = kt$

$k = 4$ $R = 500$

3) At the end of each quarter year, Rod makes a $500 payment into the Lanagham Mutual Fund. If his investments earn 7.88% annual interest compounded quarterly, what will be the value of Rod's annuity in 20 years?

APR $t = 20$ $i = \frac{.0788}{4}$ $n = 20(4) = 80$

$$FV = 500\frac{\left(1+\frac{.0788}{4}\right)^{80} - 1}{\frac{.0788}{4}}$$

$= \$95,483.39$

How much extra does he make, besides what he contributes?

$500(80) = 40,000$

$95,483 - 40,000 = \$55,483$

4) Bubba contributes $50 per month into the Lincoln National Bond Fund that earns 7.26% annual interest compounded monthly. What is the value of Bubba's investment after 25 years?

R APR $k = 12$ t

$i = \frac{.0726}{12}$ $n = 12(25) = 300$

$$FV = 50\frac{\left(1+\frac{.0726}{12}\right)^{300} - 1}{\frac{.0726}{12}}$$

$= \$42,211.46$

extra income:

$50(300) = 15,000$

$42,211 - 15,000 = \$27,211$

ANNUAL PERCENTAGE YIELD (APY):

How can you tell the difference if one bank offers an investment earning 8.75% annual interest compounded quarterly or one earning 8.7% compounded monthly?

A common basis for comparing investments is the **annual percentage yield (APY)** – the percentage rate that, compounded annually, would yield the same return as the given interest rate for the given compounding period.

5) Ursula invests $2000 with Crabby Key Bank at 5.15% annual interest compounded quarterly. What is the equivalent APY?

$A = P(1+x)$

Let x = the APY

The value of the investment at the end of one year is: $A = 2000(1 + x)$

(Recall, $A = P\left(1 + \frac{r}{k}\right)^{kt}$)

$A = 2000(1+x) = 2000\left(1 + \frac{.0515}{4}\right)^4$

$1+x = \left(1 + \frac{.0515}{4}\right)^4$

$x = \left(1 + \frac{.0515}{4}\right)^4 - 1 = .0525$

APY = 5.25%

6) Determine which investment listed at the top of the page has a higher APY.

r = 8.75% quarterly

$A = P(1+x) = P\left(1 + \frac{.0875}{4}\right)^4$

$x = \left(1 + \frac{.0875}{4}\right)^4 - 1$

$x = .0904$

9.04% APY

r = 8.7% monthly

$A = P(1+x) = P\left(1 + \frac{.087}{12}\right)^{12}$

$x = \left(1 + \frac{.087}{12}\right)^{12} - 1$

$x = .09055$

9.06% APY

Higher!