3.5B Notes Derivatives of Trigonometric Functions Name_____

1. Find the following derivatives.

u'v + v'u

a. $\frac{d}{dx}(\frac{1}{3}x^3 - \cos x)$

$x^2 + \sin x$

b. $\frac{d}{dx} x^3 \cos x$

u v

$3x^2\cos x - x^3 \sin x$

c. $\frac{d}{dx}\frac{x^3}{\tan x}$

$\frac{3x^2\tan x - x^3\sec^2 x}{\tan^2 x}$

$(\tan x)^2 = \tan^2 x$

$\neq \tan x^2$

2. Find the equation of the tangent line at the indicated point.

a. $y = \cos t, \quad t = \frac{\pi}{3}$

$y(\frac{\pi}{3}) = \cos(\frac{\pi}{3}) = \frac{1}{2}$

$y'(\frac{\pi}{3}) = -\sin(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$

b. $y = \csc x, \quad x = \frac{\pi}{4}$

$y(\frac{\pi}{4}) = \csc(\frac{\pi}{4}) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$y'(\frac{\pi}{4}) = -\csc(\frac{\pi}{4}) \cdot \cot(\frac{\pi}{4}) = -\sqrt{2} \cdot 1$

$y - \sqrt{2} = -\sqrt{2}(x - \frac{\pi}{4})$

3. Show that both $y = \cos x$ *and* $y = \sin x$ satisfy $y'' = -y$

$\cos x$

$y = \cos x$

$y' = -\sin x$

$y'' = -\cos x$

$y'' = -y$

$\sin x$

$y = \sin x$

$y' = \cos x$

$y'' = -\sin x$

$y'' = -y$

4. Calculate the first five derivatives of $f(x)=\cos x$ then determine the 8th and 37th derivatives of $f(x)=\cos x$

$f(x) = \cos x$

5 $f'(x) = -\sin x$

6 $f''(x) = -\cos x$

7 $f'''(x) = \sin x$

8 $f''''(x) = \cos x$

$f^{(5)}(x) = -\sin x$

8th derivative: $\cos x$

$f^{(8)}(x) = \frac{d^8y}{dx^8}$

37th derivative: $-\sin x$

Jerk
Jerk is the derivative of acceleration. If a body's position at time *t* is *s(t),* the body's jerk at time *t* is $$j(t)=\frac{da}{dt}=\frac{d^3s}{dt^3}$$

6. A body is moving in simple harmonic motion with position function $s=f(t)$ (s in meters, t in seconds). Find the jerk at time t given $s=2+3\sin t$.

$v = s' = 3\cos t$

$s'' = -3\sin t$

$s''' = -3\cos t$

$s(0)=2$, $s(\frac{\pi}{2})=5$, $s(\pi)=2$, $s(\frac{3\pi}{2})=-1$, $s(2\pi)=2$

$v(0)=3$, $v(\frac{\pi}{2})=0$, $v(\pi)=-3$, $v(\frac{3\pi}{2})=0$, $v(2\pi)=3$

7. Find the values of x on the interval $(0,2\pi)$ where the tangent line to the graph of $f(x)=\sin x\cos x$ is horizontal. Check your solutions on your graphing calculator.

$f'(x) = \cos x \cdot \cos x - \sin x \sin x = 0$

$\cos^2 x = \sin^2 x$

$\cos x = \pm \sin x$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$