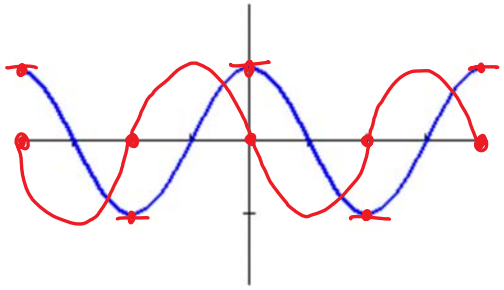
3.5 Notes

Tuesday, September 20, 2016 7:57 AM

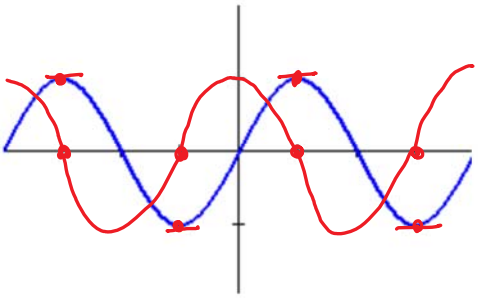
3.5A Notes Derivatives of Trigonometric Functions Name_____

1. Below is a graph of $y = \cos(x)$, graph its derivative on the same set of axes. What function is the derivative?



$\frac{dy}{dx} = -\sin x$

2. Below is a graph of $y = \sin(x)$, graph its derivative on the same set of axes. What function is the derivative?



$\frac{dy}{dx} = \cos x$

Summary

$\frac{d}{dx}\sin(x) = \cos x$ $\frac{d}{dx}\cos(x) = -\sin x$

3. Let $y = \sec(x) = \frac{1}{\cos(x)}$, using the information summarized above and the quotient rule to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\cos x(0) - 1(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$= \sec x \cdot \tan x$$

4. Let $y = \csc(x) = \frac{1}{\sin(x)}$, use the information summarized above and quotient rule to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\sin x(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= -\csc x \cdot \cot x$$

5. Let $y=\tan(x)=\frac{\sin(x)}{\cos(x)}$, use the quotient rule and the identity $\sin^2(x)+\cos^2(x)=1$ to find $\frac{dy}{dx}$.

$\frac{dy}{dx}=\frac{\cos x(\cos x)-\sin x(-\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$

$=\sec^2 x$

6. Let $y=\cot(x)=\frac{\cos(x)}{\sin(x)}$, use the quotient rule and the identity $\sin^2(x)+\cos^2(x)=1$ to find $\frac{dy}{dx}$.

$\frac{dy}{dx}=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}=\frac{-1}{\sin^2 x}$

$=-\csc^2 x$

Summary

$\frac{d}{dx}\sin(x)=\cos x$ $\quad$ $\frac{d}{dx}\cos(x)=-\sin x$ $\quad$ $\frac{d}{dx}\tan(x)=\sec^2 x$

$\frac{d}{dx}\csc(x)=-\csc x\cot x$ $\quad$ $\frac{d}{dx}\sec(x)=\sec x\tan x$ $\quad$ $\frac{d}{dx}\cot(x)=-\csc^2 x$

Important

You must memorize these 6 derivatives.

Note that the derivatives of sin(x), sec(x), and tan(x) are the only ones not preceded by a negative sign and those of the cofunctions (cos(x), csc(x), and cot(x)) are. Also note that the derivatives of the tan(x) and cot(x) are the only ones that are squares of functions. Finally, note that the derivatives of the reciprocal functions (csc(x) and sec(x)) are the only ones that are the product of two different trig functions.