Section 3.5 Exercises

For #1–18, take a logarithm of both sides of the equation, when appropriate.

1. $36\left(\frac{1}{3}\right)^{x/5} = 4$

$\left(\frac{1}{3}\right)^{x/5} = \frac{1}{9}$

$\left(\frac{1}{3}\right)^{x/5} = \left(\frac{1}{3}\right)^2$

$\frac{x}{5} = 2$

$x = 10$

2. $32\left(\frac{1}{4}\right)^{x/3} = 2$

$\left(\frac{1}{4}\right)^{x/3} = \frac{1}{16}$

$\left(\frac{1}{4}\right)^{x/3} = \left(\frac{1}{4}\right)^2$

$\frac{x}{3} = 2$

$x = 6$

3. $2 \cdot 5^{x/4} = 250$

$5^{x/4} = 125$

$5^{x/4} = 5^3$

$\frac{x}{4} = 3$

$x = 12$

4. $3 \cdot 4^{x/2} = 96$

$4^{x/2} = 32$

$4^{x/2} = 4^{5/2}$

$\frac{x}{2} = \frac{5}{2}$

$x = 5$

5. $10^{-x/3} = 10$, so $-x/3 = 1$, and therefore $x = -3$.

6. $5^{-x/4} = 5$, so $-x/4 = 1$, and therefore $x = -4$.

7. $x = 10^4 = 10{,}000$

8. $x = 2^5 = 32$

9. $x - 5 = 4^{-1}$, so $x = 5 + 4^{-1} = 5.25$.

10. $1 - x = 4^1$, so $x = -3$.

11. $x = \dfrac{\ln 4.1}{\ln 1.06} = \log_{1.06} 4.1 \approx 24.2151$

12. $x = \dfrac{\ln 1.6}{\ln 0.98} = \log_{0.98} 1.6 \approx -23.2644$

13. $e^{0.035x} = 4$, so $0.035x = \ln 4$, and therefore $x = \dfrac{1}{0.035}\ln 4 \approx 39.6084$.

14. $e^{0.045x} = 3$, so $0.045x = \ln 3$, and therefore $x = \dfrac{1}{0.045}\ln 3 \approx 24.4136$.

15. $e^{-x} = \dfrac{3}{2}$, so $-x = \ln\dfrac{3}{2}$, and therefore $x = -\ln\dfrac{3}{2} \approx -0.4055$.

16. $e^{-x} = \dfrac{5}{3}$, so $-x = \ln\dfrac{5}{3}$, and therefore $x = -\ln\dfrac{5}{3} \approx -0.5108$.

17. $\ln(x-3) = \dfrac{1}{3}$, so $x - 3 = e^{1/3}$, and therefore $x = 3 + e^{1/3} \approx 4.3956$.

18. $\log(x+2) = -2$, so $x + 2 = 10^{-2}$, and therefore $x = -2 + 10^{-2} = -1.99$.

19. We must have $x(x+1) > 0$, so $x < -1$ or $x > 0$. Domain: $(-\infty, -1) \cup (0, \infty)$; graph (e).

20. We must have $x > 0$ and $x + 1 > 0$, so $x > 0$. Domain: $(0, \infty)$; graph (f).

21. We must have $\dfrac{x}{x+1} > 0$, so $x < -1$ or $x > 0$. Domain: $(-\infty, -1) \cup (0, \infty)$; graph (d).

22. We must have $x > 0$ and $x + 1 > 0$, so $x > 0$. Domain: $(0, \infty)$; graph (c).

23. We must have $x > 0$. Domain: $(0, \infty)$; graph (a).

24. We must have $x^2 > 0$, so $x \neq 0$. Domain: $(-\infty, 0) \cup (0, \infty)$; graph (b).

For #25–38, algebraic solutions are shown (and are generally the only way to get *exact* answers). In many cases solving graphically would be faster; graphical support is also useful.

25. Write both sides as powers of 10, leaving $10^{\log x^2} = 10^6$, or $x^2 = 1{,}000{,}000$. Then $x = 1000$ or $x = -1000$.

26. Write both sides as powers of e, leaving $e^{\ln x^2} = e^4$, or $x^2 = e^4$. Then $x = e^2 \approx 7.389$ or $x = -e^2 \approx -7.389$.

27. Write both sides as powers of 10, leaving $10^{\log x^4} = 10^2$, or $x^4 = 100$. Then $x^2 = 10$, and $x = \pm\sqrt{10}$.

28. Write both sides as powers of e, leaving $e^{\ln x^6} = e^{12}$, or $x^6 = e^{12}$. Then $x^2 = e^4$, and $x = \pm e^2$.

29. Multiply both sides by $3 \cdot 2^x$, leaving $(2^x)^2 - 1 = 12 \cdot 2^x$, or $(2^x)^2 - 12 \cdot 2^x - 1 = 0$. This is quadratic in 2^x, leading to $2^x = \dfrac{12 \pm \sqrt{144 + 4}}{2} = 6 \pm \sqrt{37}$. Only $6 + \sqrt{37}$ is positive, so the only answer is $x = \dfrac{\ln(6 + \sqrt{37})}{\ln 2} = \log_2(6 + \sqrt{37}) \approx 3.5949$.

30. Multiply both sides by $2 \cdot 2^x$, leaving $(2^x)^2 + 1 = 6 \cdot 2^x$, or $(2^x)^2 - 6 \cdot 2^x + 1 = 0$. This is quadratic in 2^x, leading to $2^x = \dfrac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$. Then $x = \dfrac{\ln(3 \pm 2\sqrt{2})}{\ln 2} = \log_2(3 \pm 2\sqrt{2}) \approx \pm 2.5431$.

31. Multiply both sides by $2e^x$, leaving $(e^x)^2 + 1 = 8e^x$, or $(e^x)^2 - 8e^x + 1 = 0$. This is quadratic in e^x, leading to $e^x = \dfrac{8 \pm \sqrt{64 - 4}}{2} = 4 \pm \sqrt{15}$. Then $x = \ln(4 \pm \sqrt{15}) \approx \pm 2.0634$.

32. This is quadratic in e^x, leading to $e^x = \dfrac{-5 \pm \sqrt{25 + 24}}{4} = \dfrac{-5 \pm 7}{4}$. Of these two numbers, only $\dfrac{-5+7}{4} = \dfrac{1}{2}$ is positive, so $x = \ln\dfrac{1}{2} \approx -0.6931$.

33. $\dfrac{500}{200} = 1 + 25e^{0.3x}$, so $e^{0.3x} = \dfrac{3}{50} = 0.06$, and therefore $x = \dfrac{1}{0.3}\ln 0.06 \approx -9.3780$.

34. $\dfrac{400}{150} = 1 + 95e^{-0.6x}$, so $e^{-0.6x} = \dfrac{1}{57}$, and therefore $x = \dfrac{1}{-0.6}\ln\dfrac{1}{57} \approx 6.7384$.

35. Multiply by 2, then combine the logarithms to obtain $\ln\dfrac{x+3}{x^2} = 0$. Then $\dfrac{x+3}{x^2} = e^0 = 1$, so $x + 3 = x^2$. The solutions to this quadratic equation are $x = \dfrac{1 \pm \sqrt{1 + 12}}{2} = \dfrac{1}{2} \pm \dfrac{1}{2}\sqrt{13} \approx 2.3028$.

36. Multiply by 2, then combine the logarithms to obtain $\log\dfrac{x^2}{x+4} = 2$. Then $\dfrac{x^2}{x+4} = 10^2 = 100$, so $x^2 = 100(x+4)$. The solutions to this quadratic equation are $x = \dfrac{100 \pm \sqrt{10000 + 1600}}{2} = 50 \pm 10\sqrt{29}$. The original equation requires that $x > 0$, so $50 - 10\sqrt{29}$ is extraneous; the only actual solution is $x = 50 + 10\sqrt{29} \approx 103.8517$.

37. $\ln[(x-3)(x+4)] = 3\ln 2$, so $(x-3)(x+4) = 8$, or $x^2 + x - 20 = 0$. This factors to $(x-4)(x+5) = 0$, so $x = 4$ (an actual solution) or $x = -5$ (extraneous, since $x - 3$ and $x + 4$ must be positive).

38. $\log[(x-2)(x+5)] = 2\log 3$, so $(x-2)(x+5) = 9$, or $x^2 + 3x - 19 = 0$. Then $x = \dfrac{-3 \pm \sqrt{9 + 76}}{2} = -\dfrac{3}{2} \pm \dfrac{1}{2}\sqrt{85}$. The actual solution is $x = -\dfrac{3}{2} + \dfrac{1}{2}\sqrt{85} \approx 3.1098$; since $x - 2$ must be positive, the other algebraic solution, $x = -\dfrac{3}{2} - \dfrac{1}{2}\sqrt{85}$, is extraneous.

39. A \$100 bill has the value of 1000, or 10^3, dimes so they differ by an order of magnitude of 3.