3.4 Notes

Wednesday, September 14, 2016 11:59 AM

3.4 Velocity and Other Rates of Change

3.1

Average Rate of Change	Instantaneous Rate of Change of *f* w.r.t. *x* at *a* is the derivative:
$\dfrac{f(x+h)-f(x)}{h}$	$f'(a)=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$
over the interval *x* to *x* + *h*	**provided the limit exists**

(tangent slope)

Note: When we say" rate of change", we mean "instantaneous rate of change".

Example 1:

Find the rate of change of the volume of the sphere with respect to the length of its radius. Then, find the rate of change when the radius is 3 inches.

Start with the formula for volume of a sphere: $V=\frac{4}{3}\pi r^3$

Find the first derivative of the volume w.r.t. its radius: $\frac{dV}{dr}=\frac{4}{3}\pi(3r^2)=4\pi r^2$

Evaluate at r = 3: $\left.\frac{dV}{dr}\right|_{r=3}=4\pi(3)^2=36\pi$

Supply the correct units:

$\frac{dV}{dr}=36\pi\,\frac{in^3}{in}$

Example 2:

Find the rate of change of the $\boxed{\text{area A}}$ of a circle with respect to its radius r. Fnd the rate of change of A at r = 5 and at r = 10 (use the appropriate units)

$A=\pi r^2$

$\frac{dA}{dr}=2\pi r$

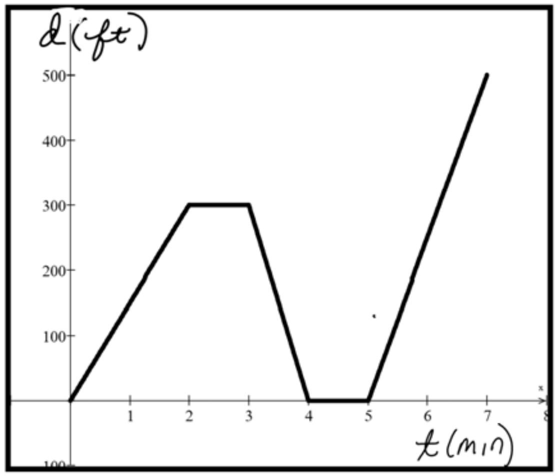
$\left.\frac{dA}{dr}\right|_{r=5}=2\pi(5)=10\pi\,\frac{in^2}{in}$

$\left.\frac{dA}{dr}\right|_{r=10}=2\pi(10)=20\pi\,\frac{in^2}{in}$

In the above problem, think of concentric circles where the radius is growing at a constant rate. Does this describe how trees grow?

Reading Position Graphs

Below is a graph of Janie's distance from home while walking to school.

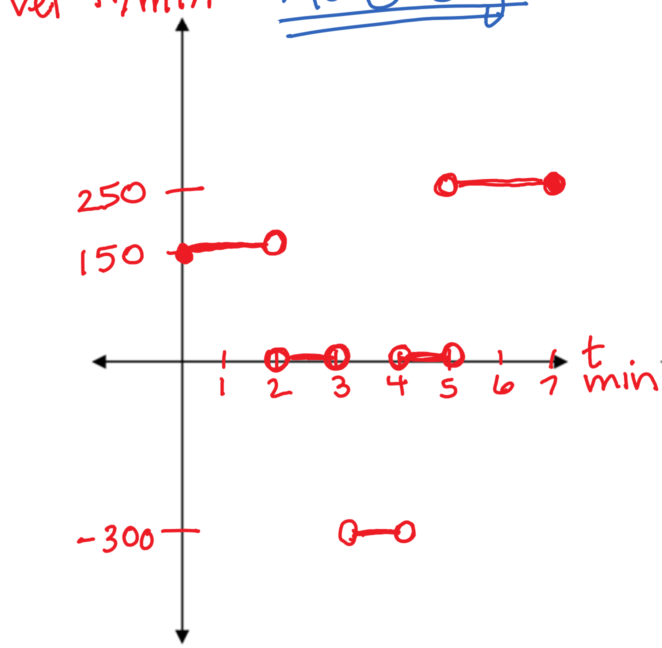


a. Describe her trip to school.

0 – 2 min walking toward school rate 150 ft/min
2 – 3 min stopped velocity = 0
3 – 4 min walking home rate −300 ft/min
4 – 5 min stopped rate = 0
5 – 7 min walking toward school. 250 ft/min

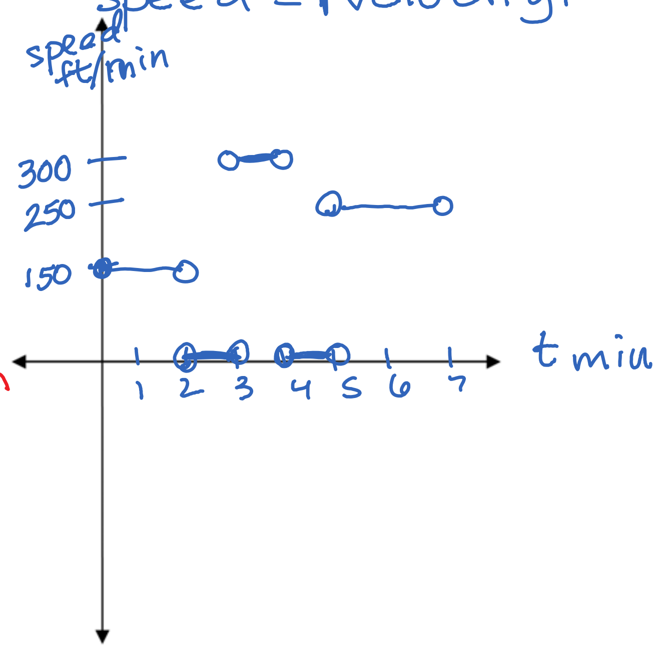
b. Sketch a graph of her velocity and her speed.

Velocity graph: has sign ±



Speed graph: all positive

speed = |velocity|



Position, Velocity, and Acceleration
Moving along a Line

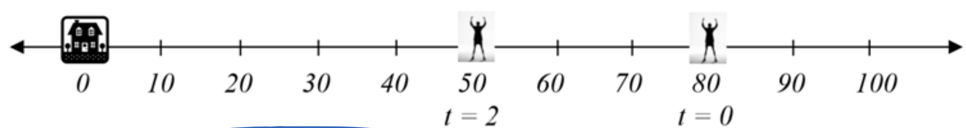
Suppose an object is moving along a linear path. We will use feet for distance and seconds for time throughout the examples that follow.

I. Let $s(t)$ be the **position** of the object at time *t*. Think of it as measuring your position compared to "home" at s(t) = 0. (Note: t = 0 is not necessarily when at "home" position.)

> ***Ex:*** *s(2)=50 means that after 2 seconds you are 50 feet ahead of home.*

- $s(b) - s(a)$ is the **displacement** during the time interval [a,b].

> ***Ex:*** *s(2) – s(0) = -30 means that you traveled 30 ft backwards during the 0-2 second time period. (So s(0) was 80ft).*



- $\dfrac{s(b)-s(a)}{b-a}$ is **avg velocity** (avg rate of change in position) during the interval [a,b]

II. $s'(t) = v(t)$ is the **(instantaneous) velocity** of the object at time t.

> ***Ex:*** $s'(5) = -20$ *means that at t = 5 seconds, you are going* <u>*backwards*</u> *at 20 ft/sec*

- $|v(t)|$ is the **speed** at time *t*. It's the same as velocity, except it ignores direction.

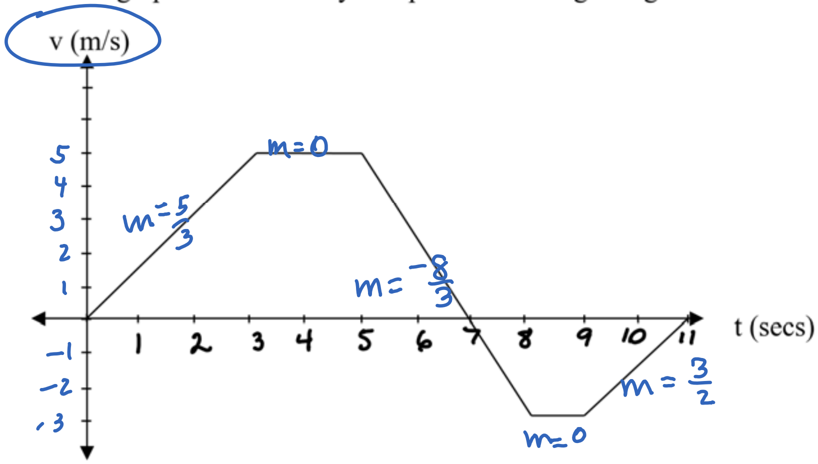
> ***Ex:*** $|s'(5)| = |v(5)| = 20$ *means you were moving 20 ft/sec at t = 5 seconds.*

III. $s''(t) = v'(t) = a(t)$ is the **acceleration** at time *t*. It measures how quickly your velocity changes.

Ex: *a(5) = -3 means that at t = 5 seconds your velocity is decreasing by 3 ft/sec*^{2}. *Since your velocity is -20 ft/sec at that time, decreasing the velocity makes it more negative, and actually makes your speed increase. You're going backwards, and you are speeding up in that direction.*

Reading Velocity Graphs

The graph below is a graph of the velocity of a particle moving along the x-axis:



a. State the intervals that the velocity is positive? Negative?

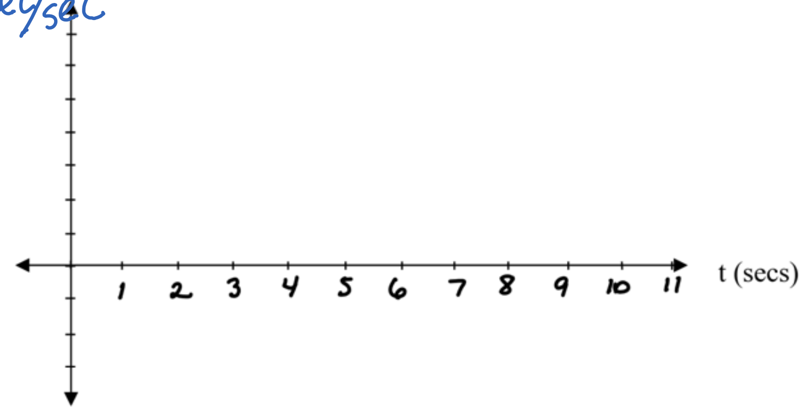
pos (0,7) neg (7,11)

velocity = 0 at t = 0, 7, 11

b. Draw a sketch of $v'(x)$ and state the intervals when it is positive? Negative?

$v'(x)$ ($a(t)$)

$m/sec^2 = m/sec/sec$



c. What is $v'(x)$ called?

d. How can we determine when the particle is speeding up and slowing down?

Modeling Vertical Motion

Distance a body released from rest falls freely is proportional to the square of the amount of time it has fallen. This is expressed as:

$$s = \frac{1}{2}gt^2$$

where *s* is the distance, *g* is the acceleration due to Earth's gravity, and *t* is time.

$g = 32$ ft/sec^2 if *s* is measured in feet
$g = 9.8$ m/sec^2 if *s* is measured in meters

Example:

On the moon, a rock is thrown vertically upward from the surface at a velocity of 10 m/sec and reaches a height of $s = 10t - 0.8t^2$. (Equation used is: $s(t) = \frac{1}{2}gt^2 + v_0t + s_0$, $g = 1.60$ m/sec^2 on moon)

a. What is the velocity of the rock at 3 seconds?

b. What is the velocity after the rock has risen 25 meters?

c. What is the maximum height of the rock? (Do not use max/min programs ☹)

d. When did the rock hit the ground?

e. How fast is the rock going when it hit the ground?

Modeling Horizontal Motion

A particle moves along a line so that its position at any time, $t \geq 0$, is given by the function $s(t) = t^3 - 12t^2 + 36t$, where *s* is measured in meters and *t* is measured in seconds.

To see this movement: MODE: parametric

$$x_{1T} = t^3 - 12t^2 + 36t$$

$$y_{1T} = 3$$

$$T:[0,10] \;\; X:[-10,50] \;\; Y:[0,5]$$

Use the ball style

a. What is the displacement of the particle during the first 2 seconds?

b. What is the average velocity of the particle during the first 2 seconds?

c. What is the (instantaneous) velocity of the particle at time *t*?

d. What is the instantaneous velocity of the particle at *t* = 4?

e. When is the particle at rest?

f. When is the particle moving in the position direction?