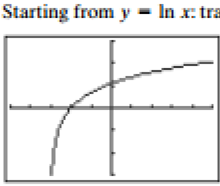
Section 3.3 Exercises

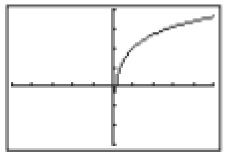
1. $\log_4 4 = 1$ because $4^1 = 4$
2. $\log_6 1 = 0$ because $6^0 = 1$
3. $\log_2 32 = 5$ because $2^5 = 32$
4. $\log_3 81 = 4$ because $3^4 = 81$
5. $\log_5 \sqrt[3]{25} = \frac{2}{3}$ because $5^{2/3} = \sqrt[3]{25}$
6. $\log_6 \frac{1}{\sqrt[5]{36}} = -\frac{2}{5}$ because $6^{-2/5} = \frac{1}{6^{2/5}} = \frac{1}{\sqrt[5]{36}}$

7. $\log 10^3 = 3$
8. $\log 10{,}000 = \log 10^4 = 4$
9. $\log 100{,}000 = \log 10^5 = 5$
10. $\log 10^{-4} = -4$
11. $\log \sqrt[3]{10} = \log 10^{1/3} = \frac{1}{3}$
12. $\log \frac{1}{\sqrt{1000}} = \log 10^{-3/2} = -\frac{3}{2}$
13. $\ln e^3 = 3$
14. $\ln e^{-4} = -4$
15. $\ln \frac{1}{e} = \ln e^{-1} = -1$
16. $\ln 1 = \ln e^0 = 0$
17. $\ln \sqrt[4]{e} = \ln e^{1/4} = \frac{1}{4}$
18. $\ln \frac{1}{\sqrt{e^7}} = \ln e^{-7/2} = -\frac{7}{2}$
19. 3, because $b^{\log_b 3} = 3$ for any $b > 0$
20. 8, because $b^{\log_b 8} = 8$ for any $b > 0$
21. $10^{\log(0.5)} = 10^{\log_{10}(0.5)} = 0.5$
22. $10^{\log 14} = 10^{\log_{10} 14} = 14$
23. $e^{\ln 6} = e^{\log_e 6} = 6$
24. $e^{\ln(1/5)} = e^{\log_e(1/5)} = 1/5$
25. $\log 9.43 \approx 0.9745 \approx 0.975$ and $10^{0.9745} \approx 9.43$
26. $\log 0.908 \approx -0.042$ and $10^{-0.042} \approx 0.908$
27. $\log(-14)$ is undefined because $-14 < 0$.
28. $\log(-5.14)$ is undefined because $-5.14 < 0$.
29. $\ln 4.05 \approx 1.399$ and $e^{1.399} \approx 4.05$
30. $\ln 0.733 \approx -0.311$ and $e^{-0.311} \approx 0.733$
31. $\ln(-0.49)$ is undefined because $-0.49 < 0$.
32. $\ln(-3.3)$ is undefined because $-3.3 < 0$.
33. $x = 10^2 = 100$
34. $x = 10^4 = 10{,}000$
35. $x = 10^{-1} = \frac{1}{10} = 0.1$
36. $x = 10^{-3} = \frac{1}{1000} = 0.001$
37. $f(x)$ is undefined for $x > 1$. The answer is (d).
38. $f(x)$ is undefined for $x < -1$. The answer is (b).
39. $f(x)$ is undefined for $x < 3$. The answer is (a).
40. $f(x)$ is undefined for $x > 4$. The answer is (c).
41. Starting from $y = \ln x$: translate left 3 units.



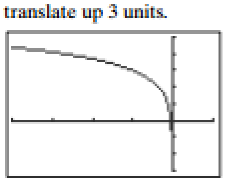
$[-5, 5]$ by $[-3, 3]$

42. Starting from $y = \ln x$: translate up 2 units.



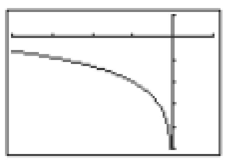
$[-5, 5]$ by $[-3, 4]$

43. Starting from $y = \ln x$: reflect across the y-axis and translate up 3 units.



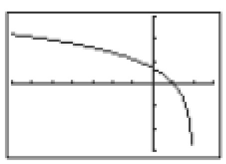
$[-4, 1]$ by $[-3, 5]$

44. Starting from $y = \ln x$: reflect across the y-axis and translate down 2 units.



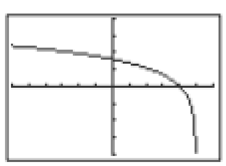
$[-4, 1]$ by $[-5, 1]$

45. Starting from $y = \ln x$: reflect across the y-axis and translate right 2 units.



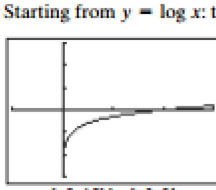
$[-7, 3]$ by $[-3, 3]$

46. Starting from $y = \ln x$: reflect across the y-axis and translate right 5 units.



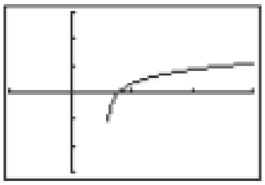
$[-6, 6]$ by $[-4, 4]$

47. Starting from $y = \log x$: translate down 1 unit.



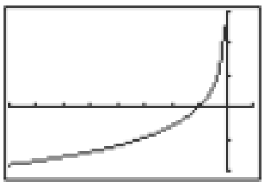
$[-5, 15]$ by $[-3, 3]$

48. Starting from $y = \log x$: translate right 3 units.



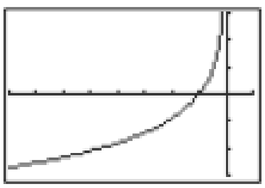
$[-5, 15]$ by $[-3, 3]$

49. Starting from $y = \log x$: reflect across both axes and vertically stretch by 2.



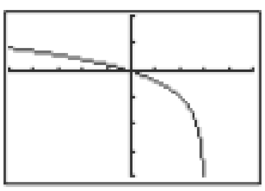
$[-8, 1]$ by $[-2, 3]$

50. Starting from $y = \log x$: reflect across both axes and vertically stretch by 3.



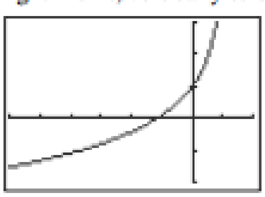
$[-8, 7]$ by $[-3, 3]$

51. Starting from $y = \log x$: reflect across the y-axis, translate right 3 units, vertically stretch by 2, translate down 1 unit.



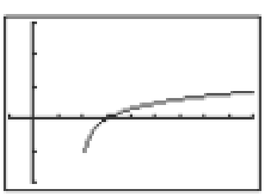
$[-5, 5]$ by $[-4, 2]$

52. Starting from $y = \log x$: reflect across both axes, translate right 1 unit, vertically stretch by 3, translate up 1 unit.



$[-6, 2]$ by $[-2, 3]$

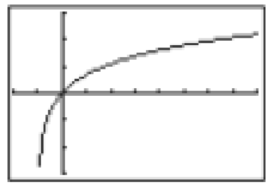
53.



$[-1, 9]$ by $[-3, 3]$

Domain: $(2, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always increasing
Not symmetric
Not bounded
No local extrema
Asymptote at $x = 2$
$\lim_{x \to \infty} f(x) = \infty$

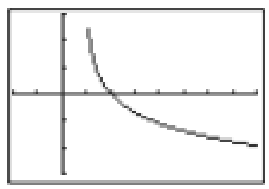
54.



$[-2, 8]$ by $[-3, 3]$

Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always increasing
Not symmetric
Not bounded
No local extrema
Asymptote: $x = -1$
$\lim_{x \to \infty} f(x) = \infty$

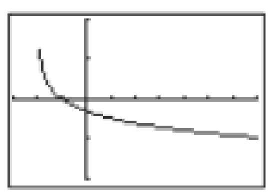
55.



$[-2, 8]$ by $[-3, 3]$

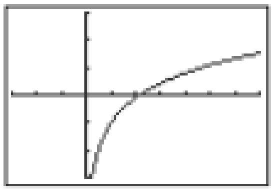
Domain: $(1, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always decreasing
Not symmetric
Not bounded
No local extrema
Asymptote: $x = 1$
$\lim_{x \to \infty} f(x) = -\infty$

56.



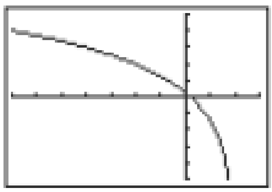
$[-3, 7]$ by $[-2, 2]$

Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always decreasing
Not symmetric
Not bounded
No local extrema
Asymptote: $x = -2$

57. 

[−3, 7] by [−3, 3]

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Continuous
Increasing on its domain
No symmetry
Not bounded
No local extrema
Asymptote at $x = 0$
$\lim_{x\to\infty} f(x) = \infty$

58. 

[−7, 3, 1] by [−10, 10, 2]

Domain: $(-\infty, 2)$
Range: $(-\infty, \infty)$
Continuous
Decreasing on its domain
No symmetry
Not bounded
No local extrema
Asymptote at $x = 2$
$\lim_{x\to-\infty} f(x) = \infty$

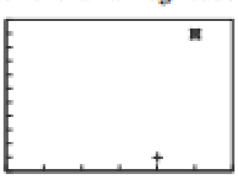
59. (a) $\beta = 10 \log\left(\frac{10^{-11}}{10^{-12}}\right) = 10 \log 10 = 10(1) = 10 \text{ dB}$

(b) $\beta = 10 \log\left(\frac{10^{-5}}{10^{-12}}\right) = 10 \log 10^{7} = 10(7) = 70 \text{ dB}$

(c) $\beta = 10 \log\left(\frac{10^{3}}{10^{-12}}\right) = 10 \log 10^{15} = 10(15) = 150 \text{ dB}$

60. $I = 12 \cdot 10^{-0.0705} \approx 10.2019$ lumens.

61. (a) A magnitude 3 earthquake is $\frac{1000}{100} = 10$ times more powerful than a magnitude 2 earthquake. A magnitude 5 earthquake is $\frac{100{,}000}{1000} = 100$ times more powerful than a magnitude 2 earthquake.

(b) 

[0, 6] by [0, 110000]

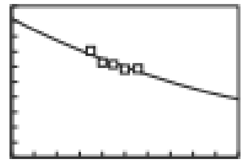
(c) Ground motion $= 10^x$ where x is the magnitude of the earthquake, so $y = 10^x$.

(d) $y = 10^x$, so $x = \log y$.

(e) Extremely large values can be represented by much smaller values.

(f) Yes

62. (a) The exponential regression model is $2552165025 \cdot 0.995838^x$, where x is the year and y is the population.

(b) 

[1900, 2100] by [100000, 1000000]

(c) Solving graphically, we find that the curve $y = 2552165025 \cdot 0.995838^x$ intersects the line $y = 500{,}000$ at $t = 2047$.

(d) Not in most cases as populations will not continue to grow without bound.

63. True, by the definition of a logarithmic function.

64. True, by the definition of common logarithm.

65. $\log 2 \approx 0.30103$. The answer is C.

66. $\log 5 \approx 0.699$ but $2.5 \log 2 \approx 0.753$. The answer is A.

67. The graph of $f(x) = \ln x$ lies entirely to the right of the origin. The answer is B.

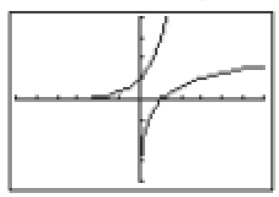
68. For $f(x) = 2 \cdot 3^x$, $f^{-1}(x) = \log_3(x/2)$ because

$$\begin{aligned} f^{-1}(f(x)) &= \log_3(2 \cdot 3^x/2) \\ &= \log_3 3^x \\ &= x. \end{aligned}$$

The answer is A.

69.

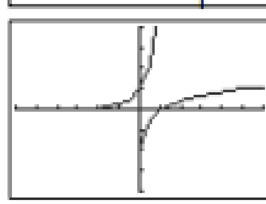
$f(x)$	3^x	$\log_3 x$
Domain	$(-\infty, \infty)$	$(0, \infty)$
Range	$(0, \infty)$	$(-\infty, \infty)$
Intercepts	$(0, 1)$	$(1, 0)$
Asymptotes	$y = 0$	$x = 0$



[−6, 6] by [−4, 4]

70.

$f(x)$	5^x	$\log_5 x$
Domain	$(-\infty, \infty)$	$(0, \infty)$
Range	$(0, \infty)$	$(-\infty, \infty)$
Intercepts	$(0, 1)$	$(1, 0)$
Asymptotes	$y = 0$	$x = 0$



[−6, 6] by [−4, 4]