5.2 Day 2 Notes

Wednesday, November 2, 2016 9:15 AM

Precalculus – 3.2B Notes
Logistic Growth Modeling

Hinsdale Central's senior class has 620 students. Seniors Halie, Katie, and Joey start a rumor that Imagine Dragons will be playing at this year's senior prom.

The rumor spreads logistically so that $S(t) = \dfrac{c}{1 + a \bullet e^{-0.9t}}$

models the number of seniors who have heard the rumor after t days.

a) Why is a logistic model more appropriate than an exponential model?

b) What is the maximum # of seniors that can hear this rumor? 620

c) Determine *c*. 620

d) How many seniors started the rumor? 3 (y-int) (not a) (0, 3)

e) Determine *a*. ____

$3 = \dfrac{620}{1 + a \cdot \cancel{e^{-.9(0)}}_{=1}}$

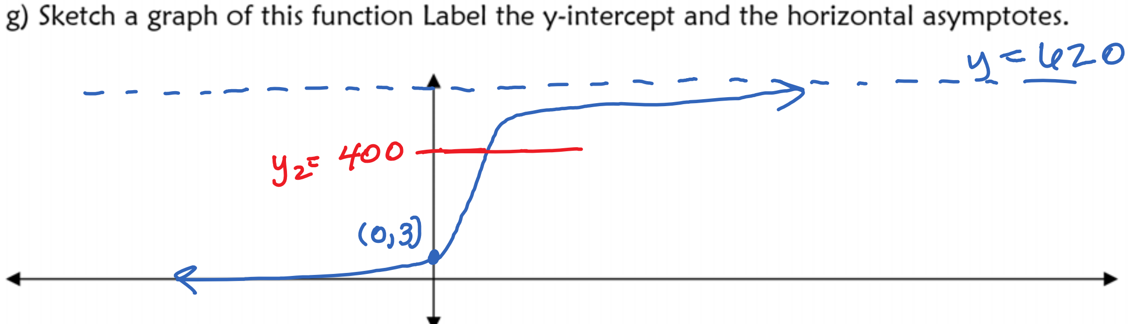
$3 = \dfrac{620}{1+a}$

$3 + 3a = 620$
$3a = 617$
$a = 205.7$

f) Write the function for S(t).

$\boxed{S(t) = \dfrac{620}{1 + 205.7e^{-.9t}}}$

g) Sketch a graph of this function Label the y-intercept and the horizontal asymptotes.



h) How long does it take for 400 students to hear the rumor? on calc!

after about t = 6.58 days

ii) How long does it take for 400 students to hear the rumor?

after about $t = 6.58$ days

Finding a Logistic Function that models the given data.

$$f(x) = \frac{c}{1 + a \bullet b^x}$$

2) Initial Value: 12 Limit to Growth: 60 Passing through (1, 24)

y-int (0,12) $c=60$ $x, f(x)$

Step 1: use y-int to solve for a

$12 = \dfrac{60}{1+a\cdot \cancel{b^0}=1}$

$12 = \dfrac{60}{1+a}$

$12+12a=60$

$12a=48$

$a=4$

Step 2: use other Given point to solve for b

$24 = \dfrac{60}{1+4\cdot b^1}$

$24 = \dfrac{60}{1+4b}$

$24+96b=60$

$96b=36$

$b=.375$

$$\boxed{f(x)=\frac{60}{1+4\cdot .375^x}}$$

3) Initial height: 5 Limit to Growth: 30 Passing through (3,15)

y-int (0,5) $c=30$

$5 = \dfrac{30}{1+a\cdot \cancel{b^0}}$

$5+5a=30$

$5a=25$

$\boxed{a=5}$

$15 = \dfrac{30}{1+5b^3}$

$15+75b^3=30$

$75b^3=15$

$b^3=.2$

$b=.585$

$$\boxed{f(x)=\frac{30}{1+5\cdot .585^x}}$$

Modeling Doubling & Half-Life

4) A culture of 100 bacteria is put into a petri dish and the culture doubles every four hours. Predict when the number of bacteria will be 350,000.

$t \approx 47$ hours

5) The $\boxed{\text{half-life}}$ of Carbon 14 is 5,730 years. If a fossil found is determined to have 15% of its original Carbon 14, how old is the fossil?

$A(t) = A_0\left(\frac{1}{2}\right)^{t/5730}$

$.15 = 1\left(\frac{1}{2}\right)^{t/5730}$

$.15 = \left(\frac{1}{2}\right)^{t/5730}$

$\log_{(\frac{1}{2})} .15 = \dfrac{t}{5730}$

$t \approx 15,683$ years