Section 3.2 Exercises

For #1–20, use the model $P(t) = P_0(1 + r)^t$.

1. $r = 0.09$, so $P(t)$ is an exponential growth function of 9%.
2. $r = 0.018$, so $P(t)$ is an exponential growth function of 1.8%.
3. $r = -0.032$, so $f(x)$ is an exponential decay function of 3.2%.
4. $r = -0.0032$, so $f(x)$ is an exponential decay function of 0.32%.
5. $r = 1$, so $g(t)$ is an exponential growth function of 100%.
6. $r = -0.95$, so $g(t)$ is an exponential decay function of 95%.
7. $f(x) = 5 \cdot (1 + 0.17)^x = 5 \cdot 1.17^x$ (x = years)
8. $f(x) = 52 \cdot (1 + 0.023)^x = 52 \cdot 1.023^x$ (x = days)
9. $f(x) = 16 \cdot (1 - 0.5)^x = 16 \cdot 0.5^x$ (x = months)
10. $f(x) = 5 \cdot (1 - 0.0059) = 5 \cdot 0.9941^x$ (x = weeks)
11. $f(x) = 28{,}900 \cdot (1 - 0.026)^x = 28{,}900 \cdot 0.974^x$ (x = years)

12. $f(x) = 502{,}000 \cdot (1 + 0.017)^x = 502{,}000 \cdot 1.017^x$ (x = years)

13. $f(x) = 18 \cdot (1 + 0.052)^x = 18 \cdot 1.052^x$ (x = weeks)

14. $f(x) = 15 \cdot (1 - 0.046)^x = 15 \cdot 0.954^x$ (x = days)

15. $f(x) = 0.6 \cdot 2^{x/3}$ (x = days)

16. $f(x) = 250 \cdot 2^{x/7.5} = 250 \cdot 2^{2x/15}$ (x = hours)

17. $f(x) = 592 \cdot 2^{-x/6}$ (x = years)

18. $f(x) = 17 \cdot 2^{-x/32}$ (x = hours)

19. $f_0 = 2.3, \dfrac{2.875}{2.3} = 1.25 = r + 1$, so $f(x) = 2.3 \cdot 1.25^x$ (Growth Model).

20. $g_0 = -5.8, \dfrac{-4.64}{-5.8} = 0.8 = r + 1$, so $g(x) = -5.8 \cdot 0.8^x$ (Decay Model).

For #21 and 22, use $f(x) = f_0 \cdot b^x$.

21. $f_0 = 4$, so $f(x) = 4 \cdot b^x$. Since $f(5) = 4 \cdot b^5 = 8.05$, $b^5 = \dfrac{8.05}{4}, b = \sqrt[5]{\dfrac{8.05}{4}} \approx 1.15.\ f(x) \approx 4 \cdot 1.15^x$.

22. $f_0 = 3$, so $f(x) = 3 \cdot b^x$. Since $f(4) = 3 \cdot b^4 = 1.49$ $b^4 = \dfrac{1.49}{3}, b = \sqrt[4]{\dfrac{1.49}{3}} \approx 0.84.\ f(x) \approx 3 \cdot 0.84^x$.

For #23–28, use the model $f(x) = \dfrac{c}{1 + a \cdot b^x}$.

23. $c = 40, a = 3$, so $f(1) = \dfrac{40}{1 + 3b} = 20, 20 + 60b = 40$, $60b = 20, b = \dfrac{1}{3}$, thus $f(x) = \dfrac{40}{1 + 3 \cdot \left(\frac{1}{3}\right)^x}$.

24. $c = 60, a = 4$, so $f(1) = \dfrac{60}{1 + 4b} = 24, 60 = 24 + 96b$, $96b = 36, b = \dfrac{3}{8}$, thus $f(x) = \dfrac{60}{1 + 4\left(\frac{3}{8}\right)^x}$.

25. $c = 128, a = 7$, so $f(5) = \dfrac{128}{1 + 7b^5} = 32$, $128 = 32 + 224b^5, 224b^5 = 96, b^5 = \dfrac{96}{224}$, $b = \sqrt[5]{\dfrac{96}{224}} \approx 0.844$, thus $f(x) \approx \dfrac{128}{1 + 7 \cdot 0.844^x}$.

26. $c = 30, a = 5$, so $f(3) = \dfrac{30}{1 + 5b^3} = 15, 30 = 15 + 75b^3$, $75b^3 = 15, b^3 = \dfrac{15}{75} = \dfrac{1}{5}, b = \sqrt[3]{\dfrac{1}{5}} \approx 0.585$, thus $f(x) \approx \dfrac{30}{1 + 5 \cdot 0.585^x}$.

27. $c = 20, a = 3$, so $f(2) = \dfrac{20}{1 + 3b^2} = 10, 20 = 10 + 30b^2$, $30b^2 = 10, b^2 = \dfrac{1}{3}, b = \sqrt{\dfrac{1}{3}} \approx 0.58$, thus $f(x) = \dfrac{20}{1 + 3 \cdot 0.58^x}$.

28. $c = 60, a = 3$, so $f(8) = \dfrac{60}{1 + 3b^8} = 30, 60 = 30 + 90b^8$, $90b^8 = 30, b^8 = \dfrac{1}{3}, b = \sqrt[8]{\dfrac{1}{3}} \approx 0.87$, thus $f(x) = \dfrac{60}{1 + 3 \cdot 0.87^x}$.

29. $P(t) = 736{,}000(1.0149)^t$; $P(t) = 1{,}000{,}000$ when $t \approx 20.73$ years, or in the year 2020.

30. $P(t) = 478{,}000(1.0628)^t$; $P(t) = 1{,}000{,}000$ when $t \approx 12.12$ years, or in the year 2012.

31. The model is $P(t) = 6250(1.0275)^t$.

(a) In 1915: about $P(25) \approx 12{,}315$. In 1940: about $P(50) \approx 24{,}265$.

(b) $P(t) = 50{,}000$ when $t \approx 76.65$ years after 1890 — in 1966.

32. The model is $P(t) = 4200(1.0225)^t$.

(a) In 1930: about $P(20) \approx 6554$. In 1945: about $P(35) \approx 9151$.

(b) $P(t) = 20{,}000$ when $t \approx 70.14$ years after 1910: about 1980.

33. **(a)** $y = 6.6\left(\dfrac{1}{2}\right)^{t/14}$, where t is time in days.

(b) After 38.11 days.

34. **(a)** $y = 3.5\left(\dfrac{1}{2}\right)^{t/65}$, where t is time in days.

(b) After 117.48 days.

35. One possible answer: Exponential and linear functions are similar in that they are always increasing or always decreasing. However, the two functions vary in how *quickly* they increase or decrease. While a linear function will increase or decrease at a steady rate over a given interval, the rate at which exponential functions increase or decrease over a given interval will vary.

36. One possible answer: Exponential functions and logistic functions are similar in the sense that they are always increasing or always decreasing. They differ, however, in the sense that logistic functions have both an upper and a lower limit to their growth (or decay), while exponential functions generally have only a lower limit. (Exponential functions just keep growing.)

37. One possible answer: From the graph we see that the doubling time for this model is 4 years. This is the time required to double from 50,000 to 100,000, from 100,000 to 200,000, or from any population size to twice that size. Regardless of the population size, it takes 4 years for it to double.

38. One possible answer: The number of atoms of a radioactive substance that change to a nonradioactive state in a given time is a fixed percentage of the number of radioactive atoms initially present. So the time it takes for half of the atoms to change state (the half-life) does not depend on the initial amount.

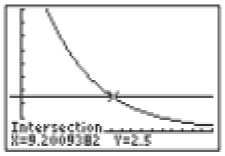
39. When $t = 1, B \approx 200$ — the population doubles every hour.

40. The half-life is about 5700 years.

For #41 and 42, use the formula $P(h) = 14.7 \cdot 0.5^{h/3.6}$, where h is miles above sea level.

41. $P(10) = 14.7 \cdot 0.5^{10/3.6} = 2.14$ lb/in²

42. $P(h) = 14.7 \cdot 0.5^{h/3.6}$ intersects $y = 2.5$ when $h \approx 9.20$ miles above sea level.



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43. The exponential regression model is $P(t) = 1149.61904(1.012133)^t$, where $P(t)$ is measured in thousands of people and t is years since 1900. The predicted population for Los Angeles for 2011 is $P(111) \approx 4384.5$, or 4,384,500 people. This is an overestimate of 565,000 people,

an error of $\frac{565{,}000}{3{,}820{,}000} \approx 0.15 = 15\%$.

44. The exponential regression model using 1950–2000 data is $P(t) = 20.84002(1.04465)^t$, where $P(t)$ is measured in thousands of people and t is years since 1900. The predicted population for Phoenix for 2011 is $P(111) \approx 2658.6$, or 2,658,600 people. This is an overestimate of 1,189,200 people,

an error of $\frac{1{,}189{,}200}{1{,}469{,}4000} \approx 0.81 = 81\%$.

The equations in #45 and 46 can be solved either algebraically or graphically; the latter approach is generally faster.

45. (a) $P(0) = 16$ students.

(b) $P(t) = 200$ when $t \approx 13.97$ — about 14 days.

(c) $P(t) = 300$ when $t \approx 16.90$ — about 17 days.

46. (a) $P(0) = 11$