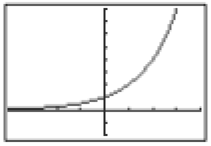
Exploration 2

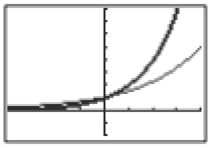
1.



$f(x) = 2^x$

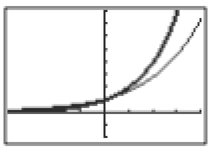
[−4, 4] by [−2, 8]

2.



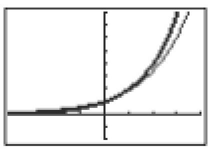
$f(x) = 2^x$
$g(x) = e^{0.4x}$

[−4, 4] by [−2, 8]



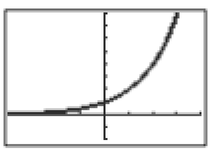
$f(x) = 2^x$
$g(x) = e^{0.5x}$

[−4, 4] by [−2, 8]



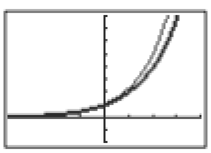
$f(x) = 2^x$
$g(x) = e^{0.6x}$

[−4, 4] by [−2, 8]



$f(x) = 2^x$
$g(x) = e^{0.7x}$

[−4, 4] by [−2, 8]



$f(x) = 2^x$
$g(x) = e^{0.8x}$

[−4, 4] by [−2, 8]

$k = 0.7$ most closely matches the graph of $f(x)$.

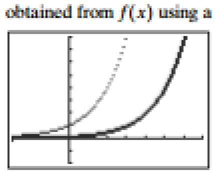
3. $k \approx 0.693$

Quick Review 3.1

1. $\sqrt[3]{-216} = -6$ since $(-6)^3 = -216$
2. $\sqrt[3]{\frac{125}{8}} = \frac{5}{2}$ since $5^3 = 125$ and $2^3 = 8$
3. $27^{2/3} = (3^3)^{2/3} = 3^2 = 9$
4. $4^{5/2} = (2^2)^{5/2} = 2^5 = 32$
5. $\frac{1}{2^{12}}$
6. $\frac{1}{3^8}$
7. $\frac{1}{a^6}$
8. b^{15}
9. -1.4, since $(-1.4)^5 = -5.37824$
10. 3.1, since $(3.1)^4 = 92.3521$

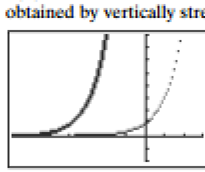
Section 3.1 Exercises

1. Not an exponential function because the base is variable and the exponent is constant. It is a monomial function.
2. Exponential function, with an initial value of 1 and base of 3.
3. Exponential function, with an initial value of 1 and base of 5.
4. Not an exponential function because the exponent is constant. It is a constant function.
5. Not an exponential function because the base is variable.
6. Not an exponential function because the base is variable. It is a power function.
7. $f(0) = 3 \cdot 5^0 = 3 \cdot 1 = 3$
8. $f(-2) = 6 \cdot 3^{-2} = \frac{6}{9} = \frac{2}{3}$
9. $f\left(\frac{1}{3}\right) = -2 \cdot 3^{1/3} = -2\sqrt[3]{3}$
10. $f(-\frac{3}{2}) = 8 \cdot 4^{-3/2} = \frac{8}{(2^2)^{3/2}} = \frac{8}{2^3} = \frac{8}{8} = 1$
11. $f(x) = \frac{3}{2} \cdot \left(\frac{1}{2}\right)^x$
12. $g(x) = 12 \cdot \left(\frac{1}{3}\right)^x$
13. $f(x) = 3 \cdot (\sqrt{2})^x = 3 \cdot 2^{x/2}$
14. $g(x) = 2 \cdot \left(\frac{1}{e}\right)^x = 2e^{-x}$
15. Translate $f(x) = 2^x$ by 3 units to the right. Alternatively, $g(x) = 2^{x-3} = 2^{-3} \cdot 2^x = \frac{1}{8} \cdot 2^x = \frac{1}{8} \cdot f(x)$, so it can be obtained from $f(x)$ using a vertical shrink by a factor of $\frac{1}{8}$.



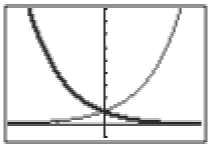
[−3, 7] by [−2, 8]

16. Translate $f(x) = 3^x$ by 4 units to the left. Alternatively, $g(x) = 3^{x+4} = 3^4 \cdot 3^x = 81 \cdot 3^x = 81 \cdot f(x)$, so it can be obtained by vertically stretching $f(x)$ by a factor of 81.



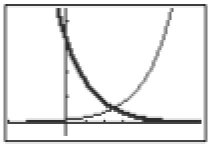
[−7, 3] by [−2, 8]

17. Reflect $f(x) = 4^x$ over the y-axis.



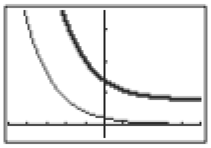
$[-2, 2]$ by $[-1, 9]$

18. Reflect $f(x) = 2^x$ over the y-axis and then shift by 5 units to the right.



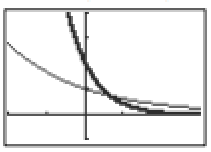
$[-3, 7]$ by $[-5, 45]$

19. Vertically stretch $f(x) = 0.5^x$ by a factor of 3 and then shift 4 units up.



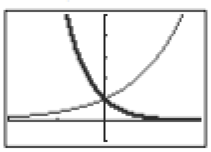
$[-5, 5]$ by $[-2, 18]$

20. Vertically stretch $f(x) = 0.6^x$ by a factor of 2 and then horizontally shrink by a factor of 3.



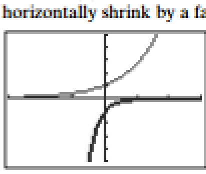
$[-2, 3]$ by $[-1, 4]$

21. Reflect $f(x) = e^x$ across the y-axis and horizontally shrink by a factor of 2.



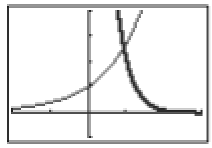
$[-2, 2]$ by $[-1, 5]$

22. Reflect $f(x) = e^x$ across the x-axis and y-axis. Then, horizontally shrink by a factor of 3.



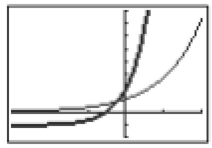
$[-3, 3]$ by $[-5, 5]$

23. Reflect $f(x) = e^x$ across the y-axis, horizontally shrink by a factor of 3, translate 1 unit to the right, and vertically stretch by a factor of 2.



$[-2, 3]$ by $[-1, 4]$

24. Horizontally shrink $f(x) = e^x$ by a factor of 2, vertically stretch by a factor of 3, and shift down 1 unit.



$[-3, 3]$ by $[-2, 8]$

25. Graph (a) is the only graph shaped and positioned like the graph of $y = b^x$, $b > 1$.

26. Graph (d) is the reflection of $y = 2^x$ across the y-axis.

27. Graph (c) is the reflection of $y = 2^x$ across the x-axis.

28. Graph (e) is the reflection of $y = 0.5^x$ across the x-axis.

29. Graph (b) is the graph of $y = 3^{-x}$ translated down 2 units.

30. Graph (f) is the graph of $y = 1.5^x$ translated down 2 units.

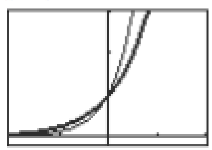
31. Exponential decay; $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to-\infty} f(x) = \infty$

32. Exponential decay; $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to-\infty} f(x) = \infty$

33. Exponential decay; $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to-\infty} f(x) = \infty$

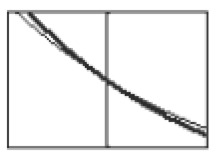
34. Exponential growth: $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to-\infty} f(x) = 0$

35. $x < 0$



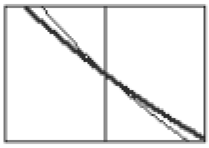
$[-2, 2]$ by $[-0.2, 3]$

36. $x > 0$



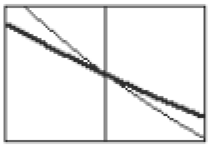
$[-0.25, 0.25]$ by $[0.5, 1.5]$

37. $x < 0$



$[-0.25, 0.25]$ by $[0.75, 1.25]$

38. $x > 0$

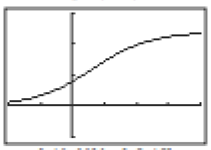


$[-0.25, 0.25]$ by $[0.75, 1.25]$

39. $y_1 = y_3$, since $3^{2x+4} = 3^{2(x+2)} = (3^2)^{x+2} = 9^{x+2}$.

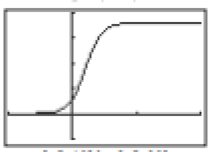
40. $y_2 = y_3$, since $2 \cdot 2^{3x-2} = 2^1\, 2^{3x-2} = 2^{1+3x-2} = 2^{3x-1}$.

41. y-intercept: $(0, 4)$. Horizontal asymptotes: $y = 0$, $y = 12$.



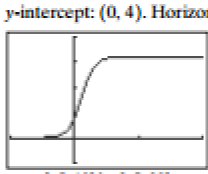
$[-10, 20]$ by $[-5, 15]$

42. y-intercept: $(0, 3)$. Horizontal asymptotes: $y = 0$, $y = 18$.



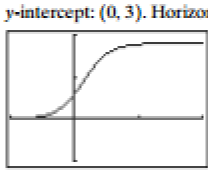
$[-5, 10]$ by $[-5, 20]$

43. y-intercept: $(0, 4)$. Horizontal asymptotes: $y = 0$, $y = 16$.



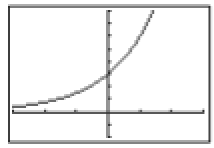
$[-5, 10]$ by $[-5, 20]$

44. y-intercept: $(0, 3)$. Horizontal asymptotes: $y = 0$, $y = 9$.



$[-5, 10]$ by $[-5, 10]$

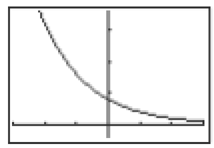
45.



$[-3, 3]$ by $[-2, 8]$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Continuous
Always increasing
Not symmetric
Bounded below by $y = 0$, which is also the only asymptote
No local extrema
$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = 0$

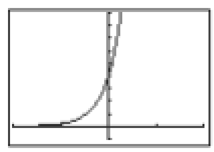
46.



$[-3, 3]$ by $[-2, 18]$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Continuous
Always decreasing
Not symmetric
Bounded below by $y = 0$, which is the only asymptote
No local extrema
$\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = \infty$

47.



$[-2, 2]$ by $[-1, 9]$

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Continuous
Always increasing
Not symmetric
Bounded below by $y = 0$, which is the only asymptote
No local extrema
$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = 0$