8. $\dfrac{x(3x-4)+(x+1)(x-1)}{(x-1)(3x-4)} = \dfrac{3x^2-4x+x^2-1}{(x-1)(3x-4)}$

$= \dfrac{4x^2-4x-1}{(x-1)(3x-4)} = \dfrac{4x^2-4x-1}{3x^2-7x+4}$

9. (a) $\dfrac{\pm1,\ \pm3}{\pm1,\ \pm2}$ or $\pm1,\ \pm\dfrac{1}{2},\ \pm3,\ \pm\dfrac{3}{2}$

(b) A graph suggests that -1 and $\dfrac{3}{2}$ are good candidates for zeros.

-1	2	1	−4	−3
		−2	1	3
$3/2$	2	−1	−3	0
		3	3	
	2	2	0	

$2x^3+x^2-4x-3 = (x+1)\left(x-\dfrac{3}{2}\right)(2x+2)$
$= (x+1)(2x-3)(x+1)$

10. (a) $\dfrac{\pm1,\ \pm2,\ \pm4,\ \pm8}{\pm1,\ \pm3}$

or $\pm1,\ \pm\dfrac{1}{3},\ \pm2,\ \pm\dfrac{2}{3},\ \pm4,\ \pm\dfrac{4}{3},\ \pm8,\ \pm\dfrac{8}{3}$

(b) A graph suggests that -2 and 1 are good candidates for zeros.

-2	3	−1	−10	8
		−6	14	−8
1	3	−7	4	0
		3	−4	
	3	−4	0	

$3x^3-x^2-10x+8 = (x+2)(x-1)(3x-4)$

Section 2.8 Exercises

1. (a) $f(x)=0$ when $x=-2,-1,5$

(b) $f(x)>0$ when $-2<x<-1$ or $x>5$

(c) $f(x)<0$ when $x<-2$ or $-1<x<5$

$(-)(-)(-)$	$(+)(-)(-)$	$(+)(+)(-)$	$(+)(+)(+)$
Negative	Positive	Negative	Positive

Critical points on the x-axis: -2, -1, 5

2. (a) $f(x)=0$ when $x=7,-\dfrac{1}{3},-4$

(b) $f(x)>0$ when $-4<x<-\dfrac{1}{3}$ or $x>7$

(c) $f(x)<0$ when $x<-4$ or $-\dfrac{1}{3}<x<7$

$(-)(-)(-)$	$(-)(-)(+)$	$(-)(+)(+)$	$(+)(+)(+)$
Negative	Positive	Negative	Positive

Critical points on the x-axis: -4, $-\frac{1}{3}$, 7

3. (a) $f(x)=0$ when $x=-7,-4,6$

(b) $f(x)>0$ when $x<-7$ or $-4<x<6$ or $x>6$

(c) $f(x)<0$ when $-7<x<-4$

$(-)(-)(-)^2$	$(+)(-)(-)^2$	$(+)(+)(-)^2$	$(+)(+)(+)^2$
Positive	Negative	Positive	Positive

Critical points on the x-axis: -7, -4, 6

4. (a) $f(x)=0$ when $x=-\dfrac{3}{5},1$

(b) $f(x)>0$ when $x<-\dfrac{3}{5}$ or $x>1$

(c) $f(x)<0$ when $-\dfrac{3}{5}<x<1$

$(-)(+)(-)$	$(+)(+)(-)$	$(+)(+)(+)$
Positive	Negative	Positive

Critical points on the x-axis: $-\frac{3}{5}$, 1

5. (a) $f(x)=0$ when $x=8,-1$

(b) $f(x)>0$ when $-1<x<8$ or $x>8$

(c) $f(x)<0$ when $x<-1$

$(+)(-)^2(-)^3$	$(+)(-)^2(+)^3$	$(+)(+)^2(+)^3$
Negative	Positive	Positive

Critical points on the x-axis: -1, 8

6. (a) $f(x)=0$ when $x=-2,9$

(b) $f(x)>0$ when $-2<x<9$ or $x>9$

(c) $f(x)<0$ when $x<-2$

$(-)^3(+)(-)^4$	$(+)^3(+)(-)^4$	$(+)^3(+)(+)^4$
Negative	Positive	Positive

Critical points on the x-axis: -2, 9

7. $(x+1)(x-3)^2=0$ when $x=-1,3$

$(-)(-)^2$	$(+)(-)^2$	$(+)(+)^2$
Negative	Positive	Positive

Critical points on the x-axis: -1, 3

By the sign chart, the solution of $(x+1)(x-3)^2>0$ is $(-1,3)\cup(3,\infty)$.

8. $(2x+1)(x-2)(3x-4)=0$ when $x=-\dfrac{1}{2},2,\dfrac{4}{3}$

$(-)(-)(-)$	$(+)(-)(-)$	$(+)(-)(+)$	$(+)(+)(+)$
Negative	Positive	Negative	Positive

Critical points on the x-axis: $-\frac{1}{2}$, $\frac{4}{3}$, 2

By the sign chart, the solution of $(2x+1)(x-2)(3x-4)\le 0$ is $\left(-\infty,-\dfrac{1}{2}\right]\cup\left[\dfrac{4}{3},2\right]$.

9. $(x+1)(x^2-3x+2)=(x+1)(x-1)(x-2)=0$ when $x=-1,1,2$

$(-)(-)(-)$	$(+)(-)(-)$	$(+)(+)(-)$	$(+)(+)(+)$
Negative	Positive	Negative	Positive

Critical points on the x-axis: -1, 1, 2

By the sign chart, the solution of $(x+1)(x-1)(x-2)<0$ is $(-\infty,-1)\cup(1,2)$.

10. $(2x-7)(x^2-4x+4)=(2x-7)(x-2)^2=0$ when $x=\dfrac{7}{2},2$

$(-)(-)^2$	$(-)(+)^2$	$(+)(+)^2$
Negative	Negative	Positive

Critical points on the x-axis: 2, $\frac{7}{2}$

By the sign chart, the solution of $(2x-7)(x-2)^2>0$ is $\left(\dfrac{7}{2},\infty\right)$.

11. By the Rational Zeros Theorem, the possible rational zeros are $\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6$. A graph suggests that $-2, \frac{1}{2}$, and 3 are good candidates to be zeros.

−2	2	−3	−11	6
		−4	14	−6
3	2	−7	3	0
		6	−3	
	2	−1	0	

$2x^3 - 3x^2 - 11x + 6 = (x + 2)(x - 3)(2x - 1) = 0$ when $x = -2, 3, \frac{1}{2}$

(−)(−)(−) Negative | −2 | (+)(−)(−) Positive | $\frac{1}{2}$ | (+)(−)(+) Negative | 3 | (+)(+)(+) Positive x

By the sign chart, the solution of $(x + 2)(x - 3)(2x - 1) \geq 0$ is $\left[-2, \frac{1}{2}\right] \cup [3, \infty)$.

12. By the Rational Zeros Theorem, the possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6$. A graph suggests that -1, 2, and 3 are good candidates to be zeros.

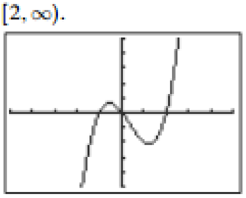
−1	1	−4	1	6
		−1	5	−6
2	1	−5	6	0
		2	−6	
	1	−3	0	

$x^3 - 4x^2 + x + 6 = (x + 1)(x - 2)(x - 3) = 0$ when $x = -1, 2, 3$.

(−)(−)(−) Negative | −1 | (+)(−)(−) Positive | 2 | (+)(+)(−) Negative | 3 | (+)(+)(+) Positive x

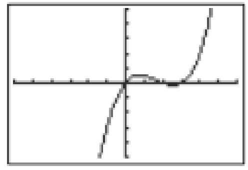
By the sign chart, the solution of $(x + 1)(x - 2)(x - 3) \leq 0$ is $(-\infty, -1] \cup [2, 3]$.

13. The zeros of $f(x) = x^3 - x^2 - 2x$ appear to be -1, 0, and 2. Substituting these values into f confirms this. The graph shows that the solution of $x^3 - x^2 - 2x \geq 0$ is $[-1, 0] \cup [2, \infty)$.



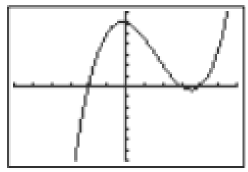
$[-5, 5]$ by $[-5, 5]$

14. The zeros of $f(x) = 2x^3 - 5x^2 + 3x$ appear to be 0, 1, and $\frac{3}{2}$. Substituting these values into f confirms this. The graph shows that the solution of $2x^3 - 5x^2 + 3x < 0$ is $(-\infty, 0) \cup \left(1, \frac{3}{2}\right)$.



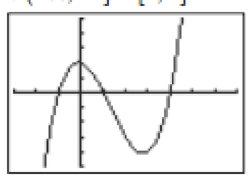
$[-3, 3]$ by $[-5, 5]$

15. The zeros of $f(x) = 2x^3 - 5x^2 - x + 6$ appear to be -1, $\frac{3}{2}$, and 2. Substituting these values into f confirms this. The graph shows that the solution of $2x^3 - 5x^2 - x + 6 > 0$ is $\left(-1, \frac{3}{2}\right) \cup (2, \infty)$.



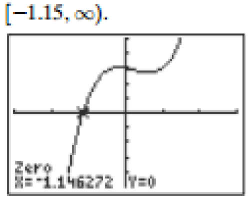
$[-3, 3]$ by $[-7, 7]$

16. The zeros of $f(x) = x^3 - 4x^2 - x + 4$ appear to be -1, 1, and 4. Substituting these values into f confirms this. The graph shows that the solution of $x^3 - 4x^2 - x + 4 \leq 0$ is $(-\infty, -1] \cup [1, 4]$.



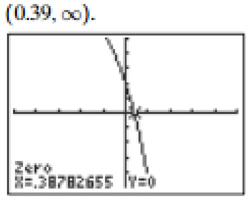
$[-3, 7]$ by $[-10, 10]$

17. The only zero of $f(x) = 3x^3 - 2x^2 - x + 6$ is found graphically to be $x \approx -1.15$. The graph shows that the solution of $3x^3 - 2x^2 - x + 6 \geq 0$ is approximately $[-1.15, \infty)$.



$[-3, 3]$ by $[-10, 10]$

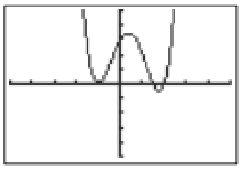
18. The only zero of $f(x) = -x^3 - 3x^2 - 9x + 4$ is found graphically to be $x \approx 0.39$. The graph shows that the solution of $-x^3 - 3x^2 - 9x + 4 < 0$ is approximately $(0.39, \infty)$.



$[-5, 5]$ by $[-10, 10]$

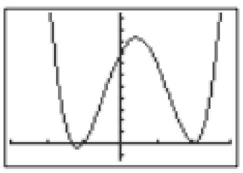
19. The zeros of $f(x) = 2x^4 - 3x^3 - 6x^2 + 5x + 6$ appear to be $-1, \frac{3}{2}$, and 2. Substituting these into f confirms this.

The graph shows that the solution of $2x^4 - 3x^3 - 6x^2 + 5x + 6 < 0$ is $\left(\frac{3}{2}, 2\right)$.



$[-5, 5]$ by $[-10, 10]$

20. The zeros of $f(x) = 3x^4 - 5x^3 - 12x^2 + 12x + 16$ appear to be $-\frac{4}{3}, -1$, and 2. Substituting these into f confirms this. The graph shows that the solution of $3x^4 - 5x^3 - 12x^2 + 12x + 16 \geq 0$ is $\left(-\infty, -\frac{4}{3}\right] \cup [-1, \infty)$.



$[-3, 3]$ by $[-3, 23]$

21. $f(x) = (x^2 + 4)(2x^2 + 3)$

(a) The solution is $(-\infty, \infty)$, because both factors of $f(x)$ are always positive.

(b) The solution is $(-\infty, \infty)$, for the same reason as in **(a)**.

(c) There are no solutions, because both factors of $f(x)$ are always positive.

(d) There are no solutions, for the same reason as in part (c).

22. $f(x) = (x^2 + 1)(-2 - 3x^2)$

(a) There are no solutions, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

(b) There are no solutions, for the same reason as in part (a).

(c) $(-\infty, \infty)$, because $x^2 + 1$ is always positive and $-2 - 3x^2$ is always negative.

(d) $(-\infty, \infty)$, for the same reason as in part (c).

23. $f(x) = (2x^2 - 2x + 5)(3x - 4)^2$
The first factor is always positive because the leading term has a positive coefficient and the discriminant $(-2)^2 - 4(2)(5) = -36$ is negative. The only zero is $x = 4/3$, with multiplicity two, since that is the solution for $3x - 4 = 0$.

(a) True for all $x \neq \frac{4}{3}$

(b) $(-\infty, \infty)$

(c) There are no solutions.

(d) $x = \frac{4}{3}$

24. $f(x) = (x^2 + 4)(3 - 2x)^2$
The first factor is always positive. The only zero is $x = 3/2$, with multiplicity two, since that is the solution for $3 - 2x = 0$.

(a) True for all $x \neq \frac{3}{2}$

(b) $(-\infty, \infty)$

(c) There are no solutions.

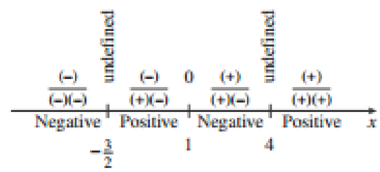
(d) $x = \frac{3}{2}$

25. **(a)** $f(x) = 0$ when $x = 1$

(b) $f(x)$ is undefined when $x = -\frac{3}{2}, 4$

(c) $f(x) > 0$ when $-\frac{3}{2} < x < 1$ or $x > 4$

(d) $f(x) < 0$ when $x < -\frac{3}{2}$ or $1 < x < 4$

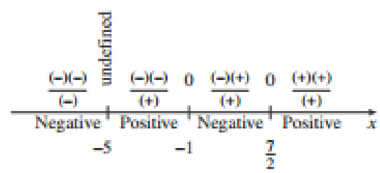


26. **(a)** $f(x) = 0$ when $x = \frac{7}{2}, -1$

(b) $f(x)$ is undefined when $x = -5$

(c) $f(x) > 0$ when $-5 < x < -1$ or $x > \frac{7}{2}$

(d) $f(x) < 0$ when $x < -5$ or $-1 < x < \frac{7}{2}$

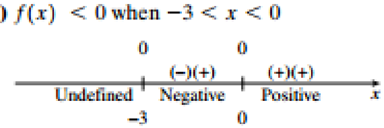


27. **(a)** $f(x) = 0$ when $x = 0, -3$

(b) $f(x)$ is undefined when $x < -3$

(c) $f(x) > 0$ when $x > 0$

(d) $f(x) < 0$ when $-3 < x < 0$

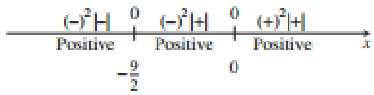


28. **(a)** $f(x) = 0$ when $x = 0, -\frac{9}{2}$

(b) None. $f(x)$ is never undefined.

(c) $f(x) > 0$ when $x \neq -\frac{9}{2}, 0$

(d) None. $f(x)$ is never negative.

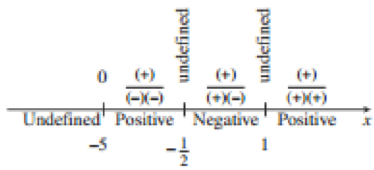


29. **(a)** $f(x) = 0$ when $x = -5$

(b) $f(x)$ is undefined when $x = -\frac{1}{2}, x = 1, x < -5$

(c) $f(x) > 0$ when $-5 < x < -\frac{1}{2}$ or $x > 1$

(d) $f(x) < 0$ when $-\frac{1}{2} < x < 1$

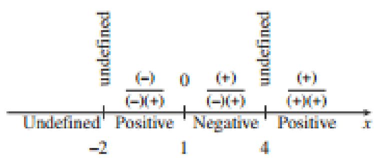


30. **(a)** $f(x) = 0$ when $x = 1$

(b) $f(x)$ is undefined when $x = 4, x \leq -2$

(c) $f(x) > 0$ when $-2 < x < 1$ or $x > 4$

(d) $f(x) < 0$ when $1 < x < 4$

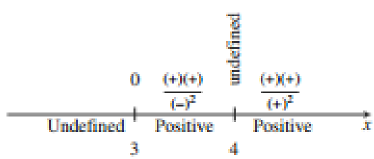


31. **(a)** $f(x) = 0$ when $x = 3$

(b) $f(x)$ is undefined when $x = 4, x < 3$

(c) $f(x) > 0$ when $3 < x < 4$ or $x > 4$

(d) None. $f(x)$ is never negative.

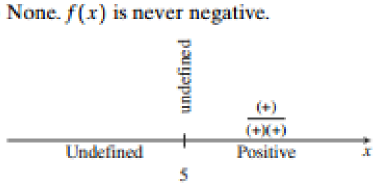


32. **(a)** None. $f(x)$is never 0.

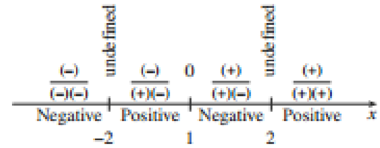
(b) $f(x)$ is undefined when $x \leq 5$

(c) $f(x) > 0$ when $5 < x < \infty$

(d) None. $f(x)$ is never negative.

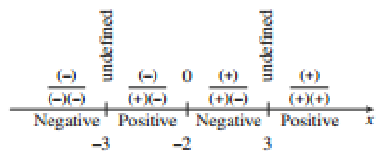


33. $f(x) = \dfrac{x - 1}{x^2 - 4} = \dfrac{x - 1}{(x + 2)(x - 2)}$ has points of potential sign change at $x = -2, 1, 2$.



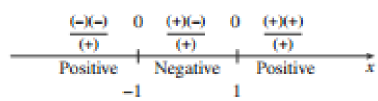
By the sign chart, the solution of $\dfrac{x - 1}{x^2 - 4} < 0$ is $(-\infty, -2) \cup (1, 2)$.

34. $f(x) = \dfrac{x + 2}{x^2 - 9} = \dfrac{x + 2}{(x + 3)(x - 3)}$ has points of potential sign change at $x = -3, -2, 3$.



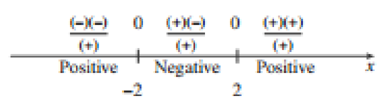
By the sign chart, the solution of $\dfrac{x + 2}{x^2 - 9} < 0$ is $(-\infty, -3) \cup (-2, 3)$.

35. $f(x) = \dfrac{x^2 - 1}{x^2 + 1} = \dfrac{(x + 1)(x - 1)}{(x^2 + 1)}$ has points of potential sign change at $x = -1, 1$.



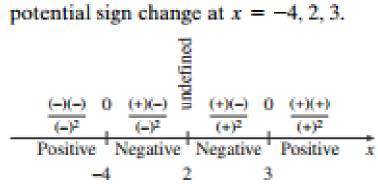
By the sign chart, the solution of $\dfrac{x^2 - 1}{x^2 + 1} \leq 0$ is $[-1, 1]$.

36. $f(x) = \dfrac{x^2 - 4}{x^2 + 4} = \dfrac{(x + 2)(x - 2)}{x^2 + 4}$ has points of potential sign change at $x = -2, 2$.



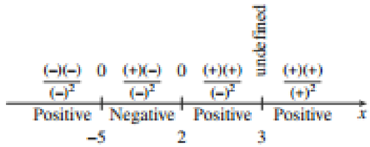
By the sign chart, the solution of $\dfrac{x^2 - 4}{x^2 + 4} > 0$ is $(-\infty, -2) \cup (2, \infty)$.

37. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4x + 4} = \dfrac{(x + 4)(x - 3)}{(x - 2)^2}$ has points of potential sign change at $x = -4, 2, 3$.



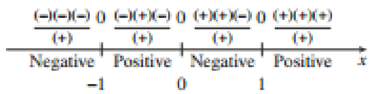
By the sign chart, the solution of $\dfrac{x^2 + x - 12}{x^2 - 4x + 4} > 0$ is $(-\infty, -4) \cup (3, \infty)$.

38. $f(x) = \dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} = \dfrac{(x+5)(x-2)}{(x-3)^2}$ has points of potential sign change at $x = -5, 2, 3$.



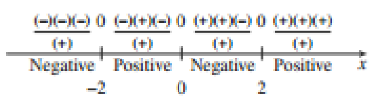
By the sign chart, the solution of $\dfrac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$ is $(-5, 2)$.

39. $f(x) = \dfrac{x^3 - x}{x^2 + 1} = \dfrac{x(x+1)(x-1)}{x^2 + 1}$ has points of potential sign change at $x = -1, 0, 1$.



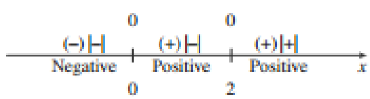
By the sign chart, the solution of $\dfrac{x^3 - x}{x^2 + 1} \geq 0$ is $[-1, 0] \cup [1, \infty)$.

40. $f(x) = \dfrac{x^3 - 4x}{x^2 + 2} = \dfrac{x(x+2)(x-2)}{x^2 + 2}$ has points of potential sign change at $x = -2, 0, 2$.



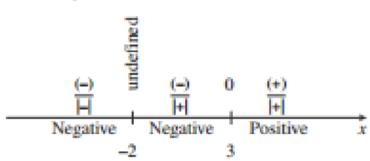
By the sign chart, the solution of $\dfrac{x^3 - 4x}{x^2 + 2} \leq 0$ is $(-\infty, -2] \cup [0, 2]$.

41. $f(x) = x|x - 2|$ has points of potential sign change at $x = 0, 2$.



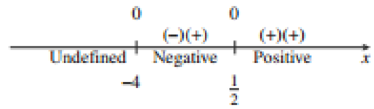
By the sign chart, the solution of $x|x - 2| > 0$ is $(0, 2) \cup (2, \infty)$.

42. $f(x) = \dfrac{x - 3}{|x + 2|}$ has points of potential sign change at $x = -2, 3$.



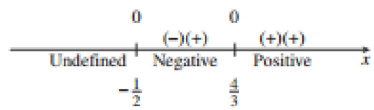
By the sign chart, the solution of $\dfrac{x - 3}{|x + 2|} < 0$ is $(-\infty, -2) \cup (-2, 3)$.

43. $f(x) = (2x - 1)\sqrt{x + 4}$ has a point of potential sign change at $x = \dfrac{1}{2}$. Note that the domain of f is $[-4, \infty)$.



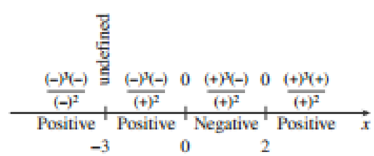
By the sign chart, the solution of $(2x - 1)\sqrt{x + 4} < 0$ is $\left(-4, \dfrac{1}{2}\right)$.

44. $f(x) = (3x - 4)\sqrt{2x + 1}$ has a point of potential sign change at $x = \dfrac{4}{3}$. Note that the domain of f is $\left[-\dfrac{1}{2}, \infty\right)$.



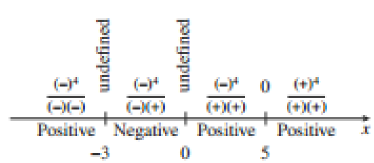
By the sign chart, the solution of $(3x - 4)\sqrt{2x + 1} \geq 0$ is $\left[\dfrac{4}{3}, \infty\right)$.

45. $f(x) = \dfrac{x^3(x - 2)}{(x + 3)^2}$ has points of potential sign change at $x = -3, 0, 2$.



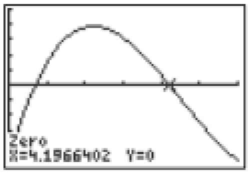
By the sign chart, the solution of $\dfrac{x^3(x - 2)}{(x + 3)^2} < 0$ is $(0, 2)$.

46. $f(x) = \dfrac{(x - 5)^4}{x(x + 3)} \geq 0$ has points of potential sign change at $x = -3, 0, 5$.



By the sign chart, the solution of $\dfrac{(x - 5)^4}{x(x + 3)} \geq 0$ is $(-\infty, -3) \cup (0, \infty)$.

59. The lengths of the sides of the box are x, $12 - 2x$, and $15 - 2x$, so the volume is $x(12 - 2x)(15 - 2x)$. To solve $x(12 - 2x)(15 - 2x) \leq 100$, graph $f(x) = x(12 - 2x)(15 - 2x) - 100$ and find the zeros: $x \approx 0.69$ and $x \approx 4.20$.



[0, 6] by [−100, 100]

From the graph, the solution of $f(x) \leq 0$ is approximately $[0, 0.69] \cup [4.20, 6]$. The squares should be such that either 0 in. $\leq x \leq$ 0.69 in. or 4.20 in. $\leq x \leq$ 6 in.