$$= \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{-2 \pm \sqrt{4 - (-24)}}{6} = \frac{-2 \pm \sqrt{28}}{6}$$
$$= \frac{-2 \pm 2\sqrt{7}}{6} = \frac{-1 \pm \sqrt{7}}{3}$$

10. For $x^2 - 3x - 9 = 0$: $a = 1, b = -3$, and $c = -9$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{9 - (-36)}}{2} = \frac{3 \pm \sqrt{45}}{2}$$
$$= \frac{3 \pm 3\sqrt{5}}{2}$$

Section 2.7 Exercises

1. Algebraically: $\frac{x-2}{3} + \frac{x+5}{3} = \frac{1}{3}$

$$(x-2) + (x+5) = 1$$
$$2x + 3 = 1$$
$$2x = -2$$
$$x = -1$$

Numerically: For $x = -1$,

$$\frac{x-2}{3} + \frac{x+5}{3} = \frac{-1-2}{3} + \frac{-1+5}{3}$$
$$= \frac{-3}{3} + \frac{4}{3}$$
$$= \frac{1}{3}.$$

2. Algebraically: $x + 2 = \frac{15}{x}$

$$x^2 + 2x = 15 \ (x \neq 0)$$
$$x^2 + 2x - 15 = 0$$
$$(x-3)(x+5) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 3 \quad \text{or} \quad x = -5$$

Numerically: For $x = 3$,
$x + 2 = 3 + 2 = 5$ and
$\frac{15}{x} = \frac{15}{3} = 5.$
For $x = -5$,
$x + 2 = -5 + 2 = -3$ and
$\frac{15}{x} = \frac{15}{-5} = -3.$

3. Algebraically: $x + 5 = \frac{14}{x}$

$$x^2 + 5x = 14 \quad (x \neq 0)$$
$$x^2 + 5x - 14 = 0$$
$$(x-2)(x+7) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x + 7 = 0$$
$$x = 2 \quad \text{or} \quad x = -7$$

Numerically: For $x = 2$,
$x + 5 = 2 + 5 = 7$ and
$\frac{14}{x} = \frac{14}{2} = 7.$
For $x = -7$,
$x + 5 = -7 + 5 = -2$ and
$\frac{14}{x} = \frac{14}{-7} = -2.$

4. Algebraically: $\frac{1}{x} - \frac{2}{x-3} = 4$

$$(x-3) - 2x = 4x(x-3) \ (x \neq 0, 3)$$
$$-x - 3 = 4x^2 - 12x$$
$$-4x^2 + 11x - 3 = 0$$
$$x = \frac{-11 \pm \sqrt{11^2 - 4(-4)(-3)}}{2(-4)}$$
$$= \frac{-11 \pm \sqrt{73}}{-8}$$
$$x = \frac{11 + \sqrt{73}}{8} \approx 2.443 \text{ or } x = \frac{11 - \sqrt{73}}{8} \approx 0.307$$

Numerically: Use a graphing calculator to support your answers numerically.

5. Algebraically: $x + \frac{4x}{x-3} = \frac{12}{x-3}$

$$x(x-3) + 4x = 12 \ (x \neq 3)$$
$$x^2 - 3x + 4x = 12$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 3 = 0$$
$x = -4$ or $x = 3$ — but $x = 3$ is extraneous.

Numerically: For $x = -4$,
$x + \frac{4x}{x-3} = -4 + \frac{4(-4)}{-4-3} = -4 + \frac{16}{7} = -\frac{12}{7}$ and
$\frac{12}{x-3} = \frac{12}{-4-3} = -\frac{12}{7}.$

6. Algebraically: $\frac{3}{x-1} + \frac{2}{x} = 8$

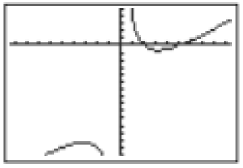
$$3x + 2(x-1) = 8x(x-1) \quad (x \neq 0, 1)$$
$$5x - 2 = 8x^2 - 8x$$
$$-8x^2 + 13x - 2 = 0$$
$$x = \frac{-13 \pm \sqrt{13^2 - 4(-8)(-2)}}{2(-8)}$$
$$= \frac{-13 \pm \sqrt{105}}{-16}$$
$$x = \frac{13 + \sqrt{105}}{16} \approx 1.453 \text{ or } x = \frac{13 - \sqrt{105}}{16} \approx 0.172$$

Numerically: Use a graphing calculator to support your answers numerically.

7. Algebraically: $x + \frac{10}{x} = 7$

$$x^2 + 10 = 7x \quad (x \neq 0)$$
$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 2 \quad \text{or} \quad x = 5$$

Graphically: The graph of $f(x) = x + \frac{10}{x} - 7$ suggests that the x-intercepts are 2 and 5.



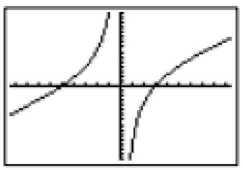
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 2$ and $x = 5$.

8. Algebraically: $x + 2 = \dfrac{15}{x}$

$$x^2 + 2x = 15 \quad (x \neq 0)$$
$$x^2 + 2x - 15 = 0$$
$$(x + 5)(x - 3) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -5 \quad \text{or} \quad x = 3$$

Graphically: The graph of $f(x) = x + 2 - \dfrac{15}{x}$ suggests that the x-intercepts are –5 and 3.



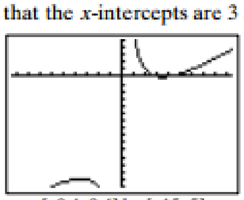
[–9.4, 9.4] by [–15, 15]

Then the solutions are $x = -5$ and $x = 3$.

9. Algebraically: $x + \dfrac{12}{x} = 7$

$$x^2 + 12 = 7x \quad (x \neq 0)$$
$$x^2 - 7x + 12 = 0$$
$$(x - 3)(x - 4) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 3 \quad \text{or} \quad x = 4$$

Graphically: The graph of $f(x) = x + \dfrac{12}{x} - 7$ suggests that the x-intercepts are 3 and 4.



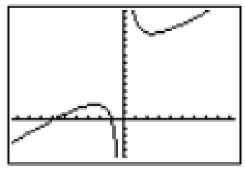
[–9.4, 9.4] by [–15, 5]

Then the solutions are $x = 3$ and $x = 4$.

10. Algebraically: $x + \dfrac{6}{x} = -7$

$$x^2 + 6 = -7x \quad (x \neq 0)$$
$$x^2 + 7x + 6 = 0$$
$$(x + 6)(x + 1) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -6 \quad \text{or} \quad x = -1$$

Graphically: The graph of $f(x) = x + \dfrac{6}{x} + 7$ suggests that the x-intercepts are –6 and –1.



[–9.4, 9.4] by [–5, 15]

Then the solutions are $x = -6$ and $x = -1$.

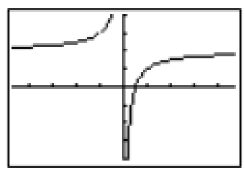
11. Algebraically: $2 - \dfrac{1}{x + 1} = \dfrac{1}{x^2 + x}$

[and $x^2 + x = x(x + 1)$]

$$2(x^2 + x) - x = 1 \quad (x \neq 0, -1)$$
$$2x^2 + x - 1 = 0$$
$$(2x - 1)(x + 1) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = \frac{1}{2} \quad \text{or} \quad x = -1$$

— but $x = -1$ is extraneous.

Graphically: The graph of $f(x) = 2 - \dfrac{1}{x + 1} - \dfrac{1}{x^2 + x}$ suggests that the x-intercept is $\dfrac{1}{2}$. There is a hole at $x = -1$.



[–4.7, 4.7] by [–4, 4]

Then the solution is $x = \dfrac{1}{2}$.

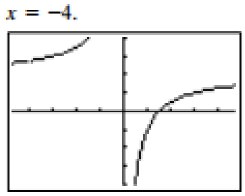
12. Algebraically: $2 - \dfrac{3}{x + 4} = \dfrac{12}{x^2 + 4x}$

[and $x^2 + 4x = x(x + 4)$]

$$2(x^2 + 4x) - 3x = 12 \quad (x \neq 0, -4)$$
$$2x^2 + 5x - 12 = 0$$
$$(2x - 3)(x + 4) = 0$$
$$2x - 3 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = \frac{3}{2} \quad \text{or} \quad x = -4$$

— but $x = -4$ is extraneous.

Graphically: The graph of $f(x) = 2 - \dfrac{3}{x + 4} - \dfrac{12}{x^2 + 4x}$ suggests that the x-intercept is $\dfrac{3}{2}$. There is a hole at $x = -4$.



[–4.7, 4.7] by [–4, 4]

Then the solution is $x = \dfrac{3}{2}$.

13. Algebraically: $\dfrac{3x}{x+5} + \dfrac{1}{x-2} = \dfrac{7}{x^2+3x-10}$

[and $x^2 + 3x - 10 = (x+5)(x-2)$]

$3x(x-2) + (x+5) = 7 \quad (x \neq -5, 2)$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

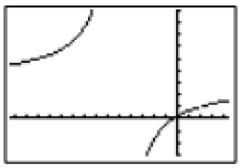
$3x + 1 = 0$ or $x - 2 = 0$

$x = -\dfrac{1}{3}$ or $x = 2$

— but $x = 2$ is extraneous.

Graphically: The graph of

$f(x) = \dfrac{3x}{x+5} + \dfrac{1}{x-2} - \dfrac{7}{x^2+3x-10}$ suggests that the x-intercept is $-\dfrac{1}{3}$. There is a hole at $x = 2$.



[–14.4, 4.4] by [–3, 9]

Then the solution is $x = -\dfrac{1}{3}$.

14. Algebraically: $\dfrac{4x}{x+4} + \dfrac{3}{x-1} = \dfrac{15}{x^2+3x-4}$

[and $x^2 + 3x - 4 = (x+4)(x-1)$]

$4x(x-1) + 3(x+4) = 15 \ (x \neq -4, 1)$

$4x^2 - x - 3 = 0$

$(4x+3)(x-1) = 0$

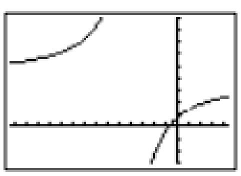
$4x + 3 = 0$ or $x - 1 = 0$

$x = -\dfrac{3}{4}$ or $x = 1$

— but $x = 1$ is extraneous.

Graphically: The graph of

$f(x) = \dfrac{4x}{x+4} + \dfrac{3}{x-1} - \dfrac{15}{x^2+3x-4}$ suggests that the x-intercept is $-\dfrac{3}{4}$. There is a hole at $x = 1$.



[–12.4, 6.4] by [–5, 10]

Then the solution is $x = -\dfrac{3}{4}$.

15. Algebraically: $\dfrac{x-3}{x} - \dfrac{3}{x+1} + \dfrac{3}{x^2+x} = 0$

[and $x^2 + x = x(x+1)$]

$(x-3)(x+1) - 3x + 3 = 0 \quad (x \neq 0, -1)$

$x^2 - 5x = 0$

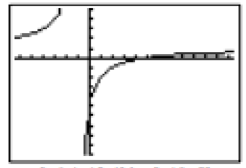
$x(x-5) = 0$

$x = 0$ or $x - 5 = 0$

$x = 0$ or $x = 5$ — but $x = 0$ is extraneous.

Graphically: The graph of

$f(x) = \dfrac{x-3}{x} - \dfrac{3}{x+1} + \dfrac{3}{x^2+x}$ suggests that the x-intercept is 5. The x-axis hides a hole at $x = 0$.



[–6.4, 12.4] by [–10, 5]

Then the solution is $x = 5$.

16. Algebraically: $\dfrac{x+2}{x} - \dfrac{4}{x-1} + \dfrac{2}{x^2-x} = 0$

[and $x^2 - x = x(x-1)$]

$(x+2)(x-1) - 4x + 2 = 0 \quad (x \neq 0, 1)$

$x^2 - 3x = 0$

$x(x-3) = 0$

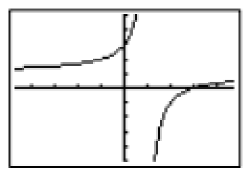
$x = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$ — but $x = 0$ is extraneous.

Graphically: The graph of

$f(x) = \dfrac{x+2}{x} - \dfrac{4}{x-1} + \dfrac{2}{x^2-x}$

suggests that the x-intercept is 3. The x-axis hides a hole at $x = 0$.



[–4.7, 4.7] by [–5, 5]

Then the solution is $x = 3$.

17. Algebraically: $\dfrac{3}{x+2} + \dfrac{6}{x^2+2x} = \dfrac{3-x}{x}$

[and $x^2 + 2x = x(x+2)$]

$3x + 6 = (3-x)(x+2) \quad (x \neq -2, 0)$

$3x + 6 = -x^2 + x + 6$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x + 2 = 0$

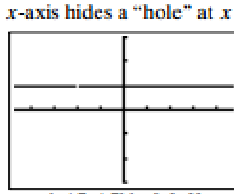
$x = 0$ or $x = -2$

— but both solutions are extraneous.

No real solutions.

Graphically: The graph of

$f(x) = \dfrac{3}{x+2} + \dfrac{6}{x^2+2x} - \dfrac{3-x}{x}$ suggests that there are no x-intercepts. There is a hole at $x = -2$, and the x-axis hides a "hole" at $x = 0$.



[–4.7, 4.7] by [–3, 3]

Then there are no real solutions.

18. Algebraically: $\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x^2+3x}$

[and $x^2 + 3x = x(x+3)$]

$(x+3)^2 - 2x = 6 \quad (x \neq -3, 0)$

$x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$x + 1 = 0$ or $x + 3 = 0$

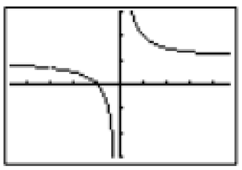
$x = -1$ or $x = -3$

— but $x = -3$ is extraneous.

Graphically: The graph of

$f(x) = \frac{x+3}{x} - \frac{2}{x+3} - \frac{6}{x^2+3x}$

suggests that the x-intercept is -1. There is a hole at $x = -3$.



[−4.7, 4.7] by [−3, 3]

Then the solution is $x = -3$.

19. There is no x-intercept at $x = -2$. That is the extraneous solution.

20. There is no x-intercept at $x = 3$. That is the extraneous solution.

21. Neither possible solution corresponds to an x-intercept of the graph. Both are extraneous.

22. There is no x-intercept at $x = 3$. That is the extraneous solution.

23. $\frac{2}{x-1} + x = 5$

$2 + x(x-1) = 5(x-1) \quad (x \neq 1)$

$x^2 - x + 2 = 5x - 5$

$x^2 - 6x + 7 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$

$x = 3 + \sqrt{2} \approx 4.414$ or

$x = 3 - \sqrt{2} \approx 1.586$

24. $\frac{x^2 - 6x + 5}{x^2 - 2} = 3$

$x^2 - 6x + 5 = 3(x^2 - 2) \quad (x \neq \pm\sqrt{2})$

$-2x^2 - 6x + 11 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-2)(11)}}{2(-2)}$

$x = \frac{6 \pm \sqrt{124}}{-4} = \frac{-3 \pm \sqrt{31}}{2}$

$x = \frac{-3 + \sqrt{31}}{2} \approx 1.284$ or

$x = \frac{-3 - \sqrt{31}}{2} \approx -4.284$

25. $\frac{x^2 - 2x + 1}{x + 5} = 0$

$x^2 - 2x + 1 = 0 \quad (x \neq 5)$

$(x - 1)^2 = 0$

$x - 1 = 0$

$x = 1$

26. $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2 + x - 2}$

[and $x^2 + x - 2 = (x+2)(x-1)$]

$3x(x-1) + 2(x+2) = 5 \quad (x \neq -2, 1)$

$3x^2 - x - 1 = 0$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{1 \pm \sqrt{13}}{6}$

$x = \frac{1 + \sqrt{13}}{6} \approx 0.768$ or

$x = \frac{1 - \sqrt{13}}{6} \approx -0.434$

27. $\frac{4x}{x+4} + \frac{5}{x-1} = \frac{15}{x^2 + 3x - 4}$

[and $x^2 + 3x - 4 = (x+4)(x-1)$]

$4x(x-1) + 5(x+4) = 15 \quad (x \neq -4, 1)$

$4x^2 + x + 5 = 0$

The discriminant is $b^2 - 4ac = 1^2 - 4(4)(5) = -79 < 0$. There are no real solutions.

28. $\frac{3x}{x+1} + \frac{5}{x-2} = \frac{15}{x^2 - x - 2}$

[and $x^2 - x - 2 = (x+1)(x-2)$]

$3x(x-2) + 5(x+1) = 15 \quad (x \neq -1, 2)$

$3x^2 - x - 10 = 0$

$(3x + 5)(x - 2) = 0$

$3x + 5 = 0$ or $x - 2 = 0$

$x = -\frac{5}{3}$ or $x = 2$

— but $x = 2$ is extraneous.

The solution is $x = -\frac{5}{3}$.

29. $x^2 + \frac{5}{x} = 8$

$x^3 + 5 = 8x \quad (x \neq 0)$

Using a graphing calculator to find the x-intercepts of $f(x) = x^3 - 8x + 5$ yields the solutions $x \approx -3.100$, $x \approx 0.661$, and $x \approx 2.439$.

30. $x^2 - \frac{3}{x} = 7$

$x^3 - 3 = 7x \quad (x \neq 0)$

Using a graphing calculator to find the x-intercepts of $f(x) = x^3 - 7x - 3$ yields the solutions $x \approx -2.398$, $x \approx -0.441$, and $x \approx 2.838$.

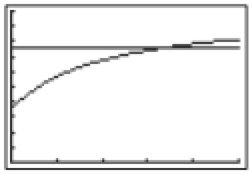
31. **(a)** The total amount of solution is $(125 + x)$ mL; of this, the amount of acid is x plus 60% of the original amount, or $x + 0.6(125)$.

(b) $y = 0.83$

(c) $C(x) = \dfrac{x + 75}{x + 125} = 0.83$. Multiply both sides by $x + 125$, then rearrange to get $0.17x = 28.75$, so that $x \approx 169.12$ mL.

32. (a) $C(x) = \dfrac{x + 0.35(100)}{x + 100} = \dfrac{x + 35}{x + 100}$

(b) Graph $C(x)$ along with $y = 0.75$; observe where the first graph intersects the second.



[0, 250] by [0, 1]

For $x = 160$, $C(x) = 0.75$. Use 160 mL.

(c) Starting from $\dfrac{x + 35}{x + 100} = 0.75$, multiply by $x + 100$ and rearrange to get $0.25x = 40$, so that $x = 160$ mL. That is how much pure acid must be added.