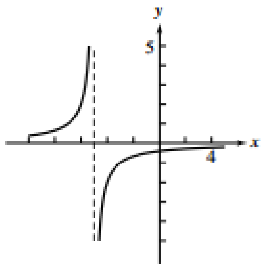
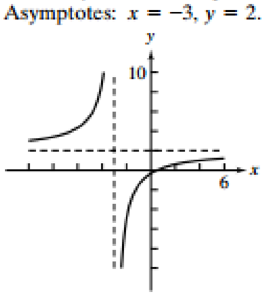
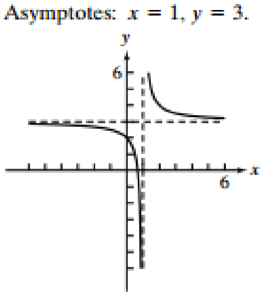
6. Translate left 5 units, reflect across x-axis, vertically stretch by 2. Asymptotes: $x = -5$, $y = 0$.



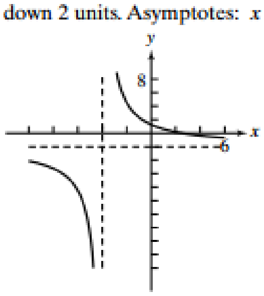
7. Translate left 3 units, reflect across x-axis, vertically stretch by 7, translate up 2 units.
Asymptotes: $x = -3$, $y = 2$.



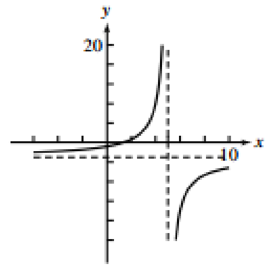
8. Translate right 1 unit, translate up 3 units.
Asymptotes: $x = 1$, $y = 3$.



9. Translate left 4 units, vertically stretch by 13, translate down 2 units. Asymptotes: $x = -4$, $y = -2$.



10. Translate right 5 units, vertically stretch by 11, reflect across x-axis, translate down 3 units. Asymptotes: $x = 5$, $y = -3$.



11. $\lim_{x \to 3^-} f(x) = \infty$

12. $\lim_{x \to 3^+} f(x) = -\infty$

13. $\lim_{x \to \infty} f(x) = 0$

14. $\lim_{x \to -\infty} f(x) = 0$

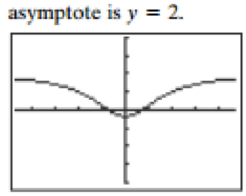
15. $\lim_{x \to -3^+} f(x) = \infty$

16. $\lim_{x \to -3^-} f(x) = -\infty$

17. $\lim_{x \to -\infty} f(x) = 5$

18. $\lim_{x \to \infty} f(x) = 5$

19. The graph of $f(x) = (2x^2 - 1)/(x^2 + 3)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 2$.



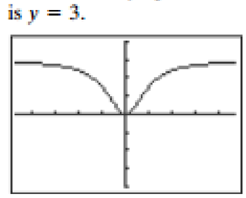
[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

$$f(x) = \frac{2x^2 - 1}{x^2 + 3} = 2 - \frac{7}{x^2 + 3}.$$

When the value of $|x|$ is large, the denominator $x^2 + 3$ is a large positive number, and $7/(x^2 + 3)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore, $\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 2$, so $y = 2$ is indeed a horizontal asymptote.

20. The graph of $f(x) = 3x^2/(x^2 + 1)$ suggests that there are no vertical asymptotes and that the horizontal asymptote is $y = 3$.



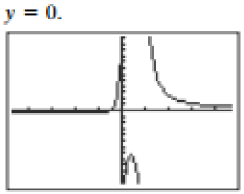
[–4.7, 4.7] by [–4, 4]

The domain of $f(x)$ is all real numbers, so there are indeed no vertical asymptotes. Using polynomial long division, we find that

$$f(x) = \frac{3x^2}{x^2+1} = 3 - \frac{3}{x^2+1}$$

When the value of $|x|$ is large, the denominator $x^2 + 1$ is a large positive number, and $3/(x^2 + 1)$ is a small positive number, getting closer to zero as $|x|$ increases. Therefore, $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$, so $y = 3$ is indeed a horizontal asymptote.

21. The graph of $f(x) = (2x + 1)/(x^2 - x)$ suggests that there are vertical asymptotes at $x = 0$ and $x = 1$, with $\lim_{x\to0^-} f(x) = \infty$, $\lim_{x\to0^+} f(x) = -\infty$, $\lim_{x\to1^-} f(x) = -\infty$, and $\lim_{x\to1^+} f(x) = \infty$, and that the horizontal asymptote is $y = 0$.



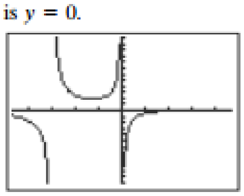
[–4.7, 4.7] by [–12, 12]

The domain of $f(x) = (2x + 1)/(x^2 - x) = (2x + 1)/[x(x - 1)]$ is all real numbers $x \neq 0, 1$, so there are indeed vertical asymptotes at $x = 0$ and $x = 1$. Rewriting one rational expression as two, we find that

$$f(x) = \frac{2x+1}{x^2-x} = \frac{2x}{x^{2-x}} + \frac{1}{x^{2-x}}$$
$$= \frac{2}{x-1} + \frac{1}{x^2-x}.$$

When the value of $|x|$ is large, both terms get close to zero. Therefore,
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$,
so $y = 0$ is indeed a horizontal asymptote.

22. The graph of $f(x) = (x - 3)/(x^2 + 3x)$ suggests that there are vertical asymptotes at $x = -3$ and $x = 0$, with $\lim_{x\to-3^-} f(x) = -\infty$, $\lim_{x\to-3^+} f(x) = \infty$, $\lim_{x\to0^-} f(x) = \infty$, and $\lim_{x\to0^+} f(x) = -\infty$, and that the horizontal asymptote is $y = 0$.



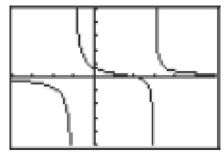
[–4.7, 4.7] by [–4, 4]

The domain of $f(x) = (x - 3)/(x^2 + 3x) = (x - 3)/[x(x + 3)]$ is all real numbers $x \neq -3, 0$, so there are indeed vertical asymptotes at $x = -3$ and $x = 0$. Rewriting one rational expression as two, we find that

$$f(x) = \frac{x-3}{x^2+3x} = \frac{x}{x^2+3x} - \frac{3}{x^2+3x}$$
$$= \frac{1}{x+3} - \frac{3}{x^2+3x}.$$

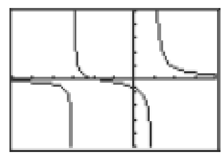
When the value of $|x|$ is large, both terms get close to zero. Therefore,
$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$,
so $y = 0$ is indeed a horizontal asymptote.

23. Intercepts: $\left(0, \frac{2}{3}\right)$ and $(2, 0)$. Asymptotes: $x = -1$, $x = 3$, and $y = 0$.



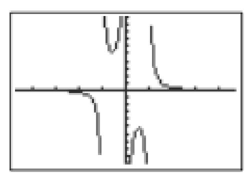
[–4, 6] by [–5, 5]

24. Intercepts: $\left(0, -\frac{2}{3}\right)$ and $(-2, 0)$. Asymptotes: $x = -3$, $x = 1$, and $y = 0$.



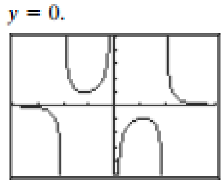
[–6, 4] by [–5, 5]

25. No intercepts. Asymptotes: $x = -1$, $x = 0$, $x = 1$, and $y = 0$.



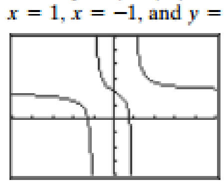
[–4.7, 4.7] by [–10, 10]

26. No intercepts. Asymptotes: $x = -2$, $x = 0$, $x = 2$, and $y = 0$.



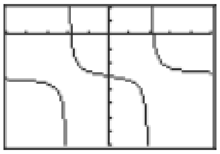
[–4, 4] by [–5, 5]

27. Intercepts: $(0, 2)$, $(-1.28, 0)$, and $(0.78, 0)$. Asymptotes: $x = 1$, $x = -1$, and $y = 2$.



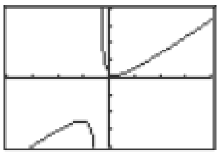
[–5, 5] by [–4, 6]

28. Intercepts: $(0, -3)$, $(-1.84, 0)$, and $(2.17, 0)$. Asymptotes: $x = -2$, $x = 2$, and $y = -3$.



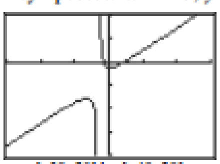
[–5, 5] by [–8, 2]

29. Intercept: $\left(0, \frac{3}{2}\right)$. Asymptotes: $x = -2$, $y = x - 4$.



[–20, 20] by [–20, 20]

30. Intercepts: $\left(0, -\frac{7}{3}\right)$, $(-1.54, 0)$, and $(4.54, 0)$.
Asymptotes: $x = -3$, $y = x - 6$.



[–30, 30] by [–40, 20]

31. (d); Xmin = –2, Xmax = 8, Xscl = 1, and Ymin = –3, Ymax = 3, Yscl = 1.

32. (b); Xmin = –6, Xmax = 2, Xscl = 1, and Ymin = –3, Ymax = 3, Yscl = 1.

33. (a); Xmin = –3, Xmax = 5, Xscl = 1, and Ymin = –5, Ymax = 10, Yscl = 1.

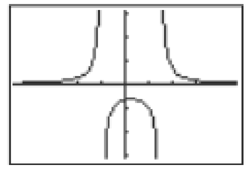
34. (f); Xmin = –6, Xmax = 2, Xscl = 1, and Ymin = –5, Ymax = 5, Yscl = 1.

35. (e); Xmin = –2, Xmax = 8, Xscl = 1, and Ymin = –3, Ymax = 3, Yscl = 1.

36. (c); Xmin = –3, Xmax = 5, Xscl = 1, and Ymin = –3, Ymax = 8, Yscl = 1.

37. For $f(x) = 2/(2x^2 - x - 3)$, the numerator is never zero, and so $f(x)$ never equals zero and the graph has no x-intercepts. Because $f(0) = -2/3$, the y-intercept is $-2/3$. The denominator factors as $2x^2 - x - 3$ $= (2x - 3)(x + 1)$, so there are vertical asymptotes at $x = -1$ and $x = 3/2$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that $\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to (3/2)^-} f(x) = -\infty$, and $\lim_{x \to (3/2)^+} f(x) = \infty$.

The graph also shows a local maximum of $-16/25$ at $x = 1/4$.



[–4.7, 4.7] by [–3.1, 3.1]

Intercept: $\left(0, -\frac{2}{3}\right)$

Domain: $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

Range: $\left(-\infty, -\frac{16}{25}\right] \cup (0, \infty)$

Continuity: All $x \neq -1, \frac{3}{2}$

Increasing on $(-\infty, -1)$ and $\left(-1, \frac{1}{4}\right]$

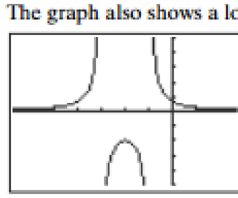
Decreasing on $\left[\frac{1}{4}, \frac{3}{2}\right)$ and $\left(\frac{3}{2}, \infty\right)$

Not symmetric
Unbounded

Local maximum at $\left(\frac{1}{4}, -\frac{16}{25}\right)$

Horizontal asymptote: $y = 0$
Vertical asymptotes: $x = -1$ and $x = 3/2$
End behavior: $\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$

38. For $g(x) = 2/(x^2 + 4x + 3)$, the numerator is never zero, and so $g(x)$ never equals zero and the graph has no x-intercepts. Because $g(0) = 2/3$, the y-intercept is $2/3$. The denominator factors as $x^2 + 4x + 3 = (x + 1)(x + 3)$, so there are vertical asymptotes at $x = -3$ and $x = -1$. And because the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. The graph supports this information and allows us to conclude that
$\lim_{x \to -3^-} g(x) = \infty$, $\lim_{x \to -3^+} g(x) = -\infty$, $\lim_{x \to -1^-} g(x) = -\infty$, and $\lim_{x \to -1^+} g(x) = \infty$.
The graph also shows a local maximum of -2 at $x = -2$.



[–6.7, 2.7] by [–5, 5]

Intercept: $\left(0, \frac{2}{3}\right)$

Domain: $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$
Range: $(-\infty, -2] \cup (0, \infty)$
Continuity: All $x \neq -3, -1$
Increasing on $(-\infty, -3)$ and $(-3, -2]$
Decreasing on $[-2, -1)$ and $(-1, \infty)$
Symmetric about $x = -2$.
Unbounded