Section 2.5 Exercises

1. $(x - 3i)(x + 3i) = x^2 - (3i)^2 = x^2 + 9$. The factored form shows the zeros to be $x = \pm 3i$. The absence of real zeros means that the graph has no x-intercepts.
2. $(x + 2)(x - \sqrt{3}\,i)(x + \sqrt{3}\,i) = (x + 2)(x^2 + 3)$
   $= x^3 + 2x^2 + 3x + 6$. The factored form shows the zeros to be $x = -2$ and $x = \pm\sqrt{3}i$. The real zero $x = -2$ is the x-intercept of the graph.
3. $(x - 1)(x - 1)(x + 2i)(x - 2i)$
   $= (x^2 - 2x + 1)(x^2 + 4)$
   $= x^4 - 2x^3 + 5x^2 - 8x + 4$. The factored form shows the zeros to be $x = 1$ (multiplicity 2) and $x = \pm 2i$. The real zero $x = 1$ is the x-intercept of the graph.
4. $x(x - 1)(x - 1 - i)(x - 1 + i)$
   $= (x^2 - x)[x - (1 + i)][x - (1 - i)]$
   $= (x^2 - x)[x^2 - (1 - i + 1 + i)\,x + (1 + 1)]$
   $= (x^2 - x)(x^2 - 2x + 2) = x^4 - 3x^3 + 4x^2 - 2x$.
   The factored form shows the zeros to be $x = 0$, $x = 1$, and $x = 1 \pm i$. The real zeros $x = 0$ and $x = 1$ are the x-intercepts of the graph.

In #5–16, any constant multiple of the given polynomial is also an answer.

5. $(x - i)(x + i) = x^2 + 1$

6. $(x - 1 + 2i)(x - 1 - 2i) = x^2 - 2x + 5$

7. $(x - 1)(x - 3i)(x + 3i) = (x - 1)(x^2 + 9)$
$= x^3 - x^2 + 9x - 9$

8. $(x + 4)(x - 1 + i)(x - 1 - i)$
$= (x + 4)(x^2 - 2x + 2) = x^3 + 2x^2 - 6x + 8$

9. $(x - 2)(x - 3)(x - i)(x + i)$
$= (x - 2)(x - 3)(x^2 + 1)$
$= x^4 - 5x^3 + 7x^2 - 5x + 6$

10. $(x + 1)(x - 2)(x - 1 + i)(x - 1 - i)$
$= (x + 1)(x - 2)(x^2 - 2x + 2)$
$= x^4 - 3x^3 + 2x^2 + 2x - 4$

11. $(x - 5)(x - 3 - 2i)(x - 3 + 2i)$
$= (x - 5)(x^2 - 6x + 13) = x^3 - 11x^2 + 43x - 65$

12. $(x + 2)(x - 1 - 2i)(x - 1 + 2i)$
$= (x + 2)(x^2 - 2x + 5) = x^3 + x + 10$

13. $(x - 1)^2(x + 2)^3 = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

14. $(x + 1)^3(x - 3) = x^4 - 6x^2 - 8x - 3$

15. $(x - 2)^2(x - 3 - i)(x - 3 + i)$
$= (x - 2)^2(x^2 - 6x + 10)$
$= (x^2 - 4x + 4)(x^2 - 6x + 10)$
$= x^4 - 10x^3 + 38x^2 - 64x + 40$

16. $(x + 1)^2(x + 2 + i)(x + 2 - i)$
$= (x + 1)^2(x^2 + 4x + 5)$
$= (x^2 + 2x + 1)(x^2 + 4x + 5)$
$= x^4 + 6x^3 + 14x^2 + 14x + 5$

In #17–20, note that the graph crosses the x-axis at odd-multiplicity zeros, and "kisses" (touches but does not cross) the x-axis where the multiplicity is even.

17. (b)

18. (c)

19. (d)

20. (a)

In #21–26, the number of complex zeros is the same as the degree of the polynomial; the number of real zeros can be determined from a graph. The latter always differs from the former by an even number (when the coefficients of the polynomial are real).

21. 2 complex zeros; none real.

22. 3 complex zeros; all 3 real.

23. 3 complex zeros; 1 real.

24. 4 complex zeros; 2 real.

25. 4 complex zeros; 2 real.

26. 5 complex zeros; 1 real.

In #27–32, look for real zeros using a graph (and perhaps the Rational Zeros Test). Use synthetic division to factor the polynomial into one or more linear factors and a quadratic factor. Then use the quadratic formula to find complex zeros.

27. Inspection of the graph reveals that $x = 1$ is the only real zero. Dividing $f(x)$ by $x - 1$ leaves $x^2 + x + 5$ (below). The quadratic formula gives the remaining zeros of $f(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 4 & -5 \\ & & 1 & 1 & 5 \\ \hline & 1 & 1 & 5 & 0 \end{array}$$

Zeros: $x = 1,\ x = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i$

$f(x)$
$= (x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{19}}{2}i\right)\right]\left[x - \left(-\frac{1}{2} + \frac{\sqrt{19}}{2}i\right)\right]$
$= \frac{1}{4}(x - 1)(2x + 1 + \sqrt{19}\,i)(2x + 1 - \sqrt{19}\,i)$

28. Zeros: $x = 3$ (graphically) and $x = \frac{7}{2} \pm \frac{\sqrt{43}}{2}i$ (applying the quadratic formula to $x^2 - 7x + 23$).

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$f(x)$
$= (x - 3)\left[x - \left(\frac{7}{2} - \frac{\sqrt{43}}{2}i\right)\right]\left[x - \left(\frac{7}{2} + \frac{\sqrt{43}}{2}i\right)\right]$
$= \frac{1}{4}(x - 3)(2x - 7 + \sqrt{43}\,i)(2x - 7 - \sqrt{43}\,i)$

29. Zeros: $x = \pm 1$(graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$ (applying the quadratic formula to $x^2 + x + 6$).

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 5 & -1 & -6 \\ & & 1 & 2 & 7 & 6 \\ \hline & 1 & 2 & 7 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 7 & 6 \\ & & -1 & -1 & -6 \\ \hline & 1 & 1 & 6 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{23}}{2}i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{23}}{2}i\right)\right]$
$= \frac{1}{4}(x - 1)(x + 1)(2x + 1 + \sqrt{23}\,i)\,(2x + 1 - \sqrt{23}\,i)$

30. Zeros: $x = -2$ and $x = \frac{1}{3}$ (graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (applying the quadratic formula to $3x^2 + 3x + 3 = 3(x^2 + x + 1)$).

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & 6 & 3 & -2 \\ & & -6 & -4 & -4 & 2 \\ \hline & 3 & 2 & 2 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1/3 & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

$f(x) = (x + 2)(3x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]$