In #5–16, any constant multiple of the given polynomial is also an answer.

5. $(x - i)(x + i) = x^2 + 1$

6. $(x - 1 + 2i)(x - 1 - 2i) = x^2 - 2x + 5$

7. $(x - 1)(x - 3i)(x + 3i) = (x - 1)(x^2 + 9)$
$= x^3 - x^2 + 9x - 9$

8. $(x + 4)(x - 1 + i)(x - 1 - i)$
$= (x + 4)(x^2 - 2x + 2) = x^3 + 2x^2 - 6x + 8$

9. $(x - 2)(x - 3)(x - i)(x + i)$
$= (x - 2)(x - 3)(x^2 + 1)$
$= x^4 - 5x^3 + 7x^2 - 5x + 6$

10. $(x + 1)(x - 2)(x - 1 + i)(x - 1 - i)$
$= (x + 1)(x - 2)(x^2 - 2x + 2)$
$= x^4 - 3x^3 + 2x^2 + 2x - 4$

11. $(x - 5)(x - 3 - 2i)(x - 3 + 2i)$
$= (x - 5)(x^2 - 6x + 13) = x^3 - 11x^2 + 43x - 65$

12. $(x + 2)(x - 1 - 2i)(x - 1 + 2i)$
$= (x + 2)(x^2 - 2x + 5) = x^3 + x + 10$

13. $(x - 1)^2(x + 2)^3 = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

14. $(x + 1)^3(x - 3) = x^4 - 6x^2 - 8x - 3$

15. $(x - 2)^2(x - 3 - i)(x - 3 + i)$
$= (x - 2)^2(x^2 - 6x + 10)$
$= (x^2 - 4x + 4)(x^2 - 6x + 10)$
$= x^4 - 10x^3 + 38x^2 - 64x + 40$

16. $(x + 1)^2(x + 2 + i)(x + 2 - i)$
$= (x + 1)^2(x^2 + 4x + 5)$
$= (x^2 + 2x + 1)(x^2 + 4x + 5)$
$= x^4 + 6x^3 + 14x^2 + 14x + 5$

In #17–20, note that the graph crosses the x-axis at odd-multiplicity zeros, and "kisses" (touches but does not cross) the x-axis where the multiplicity is even.

17. (b)

18. (c)

19. (d)

20. (a)

In #21–26, the number of complex zeros is the same as the degree of the polynomial; the number of real zeros can be determined from a graph. The latter always differs from the former by an even number (when the coefficients of the polynomial are real).

21. 2 complex zeros; none real.

22. 3 complex zeros; all 3 real.

23. 3 complex zeros; 1 real.

24. 4 complex zeros; 2 real.

25. 4 complex zeros; 2 real.

26. 5 complex zeros; 1 real.

In #27–32, look for real zeros using a graph (and perhaps the Rational Zeros Test). Use synthetic division to factor the polynomial into one or more linear factors and a quadratic factor. Then use the quadratic formula to find complex zeros.

27. Inspection of the graph reveals that $x = 1$ is the only real zero. Dividing $f(x)$ by $x - 1$ leaves $x^2 + x + 5$ (below). The quadratic formula gives the remaining zeros of $f(x)$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 4 & -5 \\ & & 1 & 1 & 5 \\ \hline & 1 & 1 & 5 & 0 \end{array}$$

Zeros: $x = 1$, $x = -\frac{1}{2} \pm \frac{\sqrt{19}}{2}i$

$f(x)$
$= (x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{19}}{2}i\right)\right]\left[x - \left(-\frac{1}{2} + \frac{\sqrt{19}}{2}i\right)\right]$
$= \frac{1}{4}(x - 1)(2x + 1 + \sqrt{19}\,i)(2x + 1 - \sqrt{19}\,i)$

28. Zeros: $x = 3$ (graphically) and $x = \frac{7}{2} \pm \frac{\sqrt{43}}{2}i$ (applying the quadratic formula to $x^2 - 7x + 23$).

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$f(x)$
$= (x - 3)\left[x - \left(\frac{7}{2} - \frac{\sqrt{43}}{2}i\right)\right]\left[x - \left(\frac{7}{2} + \frac{\sqrt{43}}{2}i\right)\right]$
$= \frac{1}{4}(x - 3)(2x - 7 + \sqrt{43}\,i)(2x - 7 - \sqrt{43}\,i)$

29. Zeros: $x = \pm 1$(graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$ (applying the quadratic formula to $x^2 + x + 6$).

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 5 & -1 & -6 \\ & & 1 & 2 & 7 & 6 \\ \hline & 1 & 2 & 7 & 6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 7 & 6 \\ & & -1 & -1 & -6 \\ \hline & 1 & 1 & 6 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{23}}{2}i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{23}}{2}i\right)\right]$
$= \frac{1}{4}(x - 1)(x + 1)(2x + 1 + \sqrt{23}\,i)\,(2x + 1 - \sqrt{23}\,i)$

30. Zeros: $x = -2$ and $x = \frac{1}{3}$ (graphically) and $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (applying the quadratic formula to $3x^2 + 3x + 3 = 3(x^2 + x + 1)$).

$$\begin{array}{r|rrrrr} -2 & 3 & 8 & 6 & 3 & -2 \\ & & -6 & -4 & -4 & 2 \\ \hline & 3 & 2 & 2 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1/3 & 3 & 2 & 2 & -1 \\ & & 1 & 1 & 1 \\ \hline & 3 & 3 & 3 & 0 \end{array}$$

$f(x) = (x + 2)(3x - 1)\left[x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right]$
$\left[x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right]$

$= \frac{1}{4}(x + 2)(3x - 1)(2x + 1 + \sqrt{3}\,i)$
$(2x + 1 - \sqrt{3}\,i)$

31. Zeros: $x = -\frac{7}{3}$ and $x = \frac{3}{2}$ (graphically) and $x = 1 \pm 2i$ (applying the quadratic formula to $6x^2 - 12x + 30 = 6(x^2 - 2x + 5)$).

$-7/3$	6	-7	-1	67	-105
		-14	49	-112	105
	6	-21	48	-45	0

$3/2$	6	-21	48	-45
		9	-18	45
	6	-12	30	0

$f(x) = (3x + 7)(2x - 3)[x - (1 - 2i)]$
$[x - (1 + 2i)]$
$= (3x + 7)(2x - 3)(x - 1 + 2i)(x - 1 - 2i)$

32. Zeros: $x = -\frac{3}{5}$ and $x = 5$ (graphically) and $x = \frac{3}{2} \pm i$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$
(applying the quadratic formula to $20x^2 - 60x + 65 = 5(4x^2 - 12x + 13)$).

5	20	-148	269	-106	-195
		100	-240	145	195
	20	-48	29	39	0

$-3/5$	20	-48	29	39
		-12	36	-39
	20	-60	65	0

$f(x) = (5x + 3)(x - 5)[2x - (3 - 2i)]$
$[2x - (3 + 2i)]$
$= (5x + 3)(x - 5)(2x - 3 + 2i)(2x - 3 - 2i)$

In #33–36, since the polynomials' coefficients are real, for the given zero $z = a + bi$, the complex conjugate $\bar{z} = a - bi$ must also be a zero. Divide $f(x)$ by $x - z$ and $x - \bar{z}$ to reduce to a quadratic.

33. First divide $f(x)$ by $x - (1 + i)$ (synthetically). Then divide the result, $x^3 + (-1 + i)x^2 - 3x + (3 - 3i)$, by $x - (1 - i)$. This leaves the polynominal $x^2 - 3$. Zeros: $x = \pm\sqrt{3}, x = 1 \pm i$

$1 + i$	1	-2	-1	6	-6
		$1 + i$	-2	$-3 - 3i$	6
	1	$-1 + i$	-3	$3 - 3i$	0

$1 - i$	1	$-1 + i$	-3	$3 - 3i$
		$1 - i$	0	$-3 + 3i$
	1	0	-3	0

$f(x) = (x - \sqrt{3})(x + \sqrt{3})[x - (1 - i)][x - (1 + i)]$
$= (x - \sqrt{3})(x + \sqrt{3})(x - 1 + i)(x - 1 - i)$

34. First divide $f(x)$ by $x - 4i$. Then divide the result, $x^3 + 4ix^2 - 3x - 12i$, by $x + 4i$. This leaves the polynomial $x^2 - 3$. Zeros: $x = \pm\sqrt{3}, x = \pm 4i$

$4i$	1	0	13	0	-48
		$4i$	-16	$-12i$	48
	1	$4i$	-3	$-12i$	0

$-4i$	1	$4i$	-3	$-12i$
		$-4i$	0	$12i$
	1	0	-3	0

$f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - 4i)(x + 4i)$

35. First divide $f(x)$ by $x - (3 - 2i)$. Then divide the result, $x^3 + (-3 - 2i)x^2 - 2x + 6 + 4i$, by $x - (3 + 2i)$. This leaves $x^2 - 2$. Zeros: $x = \pm\sqrt{2}, x = 3 \pm 2i$

$3 - 2i$	1	-6	11	12	-26
		$3 - 2i$	-13	$-6 + 4i$	26
	1	$-3 - 2i$	-2	$6 + 4i$	0

$3 + 2i$	1	$-3 - 2i$	-2	$6 + 4i$
		$3 + 2i$	0	$-6 - 4i$
	1	0	-2	0

$f(x) = (x - \sqrt{2})(x + \sqrt{2})[x - (3 - 2i)]$
$[x - (3 + 2i)]$
$= (x - \sqrt{2})(x + \sqrt{2})(x - 3 + 2i)(x - 3 - 2i)$

36. First divide $f(x)$ by $x - (1 + 3i)$. Then divide the result, $x^3 + (-1 + 3i)x^2 - 5x + 5 - 15i$, by $x - (1 - 3i)$. This leaves $x^2 - 5$. Zeros: $x = \pm\sqrt{5}, x = 1 \pm 3i$

$1 + 3i$	1	-2	5	10	-50
		$1 + 3i$	-10	$-5 - 15i$	50
	1	$-1 + 3i$	-5	$5 - 15i$	0

$1 - 3i$	1	$-1 + 3i$	-5	$5 - 15i$
		$1 - 3i$	0	$-5 + 15i$
	1	0	-5	0

$f(x) = (x - \sqrt{5})(x + \sqrt{5})[x - (1 - 3i)]$
$[x - (1 + 3i)]$
$= (x - \sqrt{5})(x + \sqrt{5})(x - 1 + 3i)(x - 1 - 3i)$

For #37–42, find real zeros graphically, then use synthetic division to find the quadratic factors. Only the synthetic divison step is shown.

37. $f(x) = (x - 2)(x^2 + x + 1)$

38. $f(x) = (x - 2)(x^2 + x + 3)$

2	1	-1	-1	-2
		2	2	2
	1	1	1	0

39. $f(x) = (x - 1)(2x^2 + x + 3)$

2	1	-1	1	-6
		2	2	6
	1	1	3	0

40. $f(x) = (x - 1)(3x^2 + x + 2)$

1	2	-1	2	-3
		2	1	3
	2	1	3	0

1	3	-2	1	-2
		3	1	2
	3	1	2	0