2.4 More Problems

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9:44 PM

Opener

① $f(x) = 3 + 2x^2$

ⓐ avg Rate of Change over interval $[1, 3]$ → slope of secant line between $(3, 21)$ and $(1, 5)$

$$\text{avg} = \frac{21-5}{3-1} = \frac{16}{2} = 8$$

ⓑ Instantaneous Rate of change at $x=3$ → slope of tangent line at $x=3$

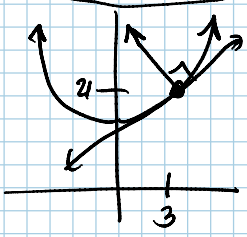
$$m = \lim_{h\to 0} \frac{3+2(3+h)^2 - (3+2(3)^2)}{h} = \lim_{h\to 0} \frac{3+2(9+6h+h^2) - 21}{h}$$

$$= \lim_{h\to 0} \frac{21+12h+2h^2-21}{h} = \lim_{h\to 0} 12+2h = \boxed{12}$$

ⓒ Eq. of Tan Line & Normal Line at $x=3$

$\boxed{y-21 = 12(x-3)}$ $\qquad$ $\boxed{y-21 = -\frac{1}{12}(x-3)}$

ⓓ Draw



(Together)

② What is rate of change of area of a rectangle if its $L = x+5$ and width $= x$ at $x = 4$ in?

$A = LW = x(x+5) = \boxed{x^2+5x}$ $\qquad$ a $f(a)$ $(4, 36)$

$$m = \lim_{h\to 0} \frac{(4+h)^2+5(4+h)-(36)}{h} = \lim_{h\to 0} \frac{16+8h+h^2+20+5h-36}{h}$$

$$= \lim_{h\to 0} \frac{13h+h^2}{h} = \lim_{h\to 0} 13+h = \boxed{13 \text{ in}^2/\text{in}}$$ ← what does this mean?

for every inch x grows, area is growing by 13in^2 at $x=4$.

③ $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ what is rate of change of Volume w/respect to radius when $r = 2$ in? $\qquad$ $(2, \frac{32\pi}{3})$

$$m = \lim_{h\to 0} \frac{\frac{4}{3}\pi(2+h)^3 - \frac{32\pi}{3}}{h} = \lim \frac{\frac{4}{3}\pi(8+12h+6h^2+h^3) - \frac{32\pi}{3}}{}$$

$$m = \lim_{h\to 0} \frac{\frac{4}{3}\pi(2+h)^3 - \frac{32\pi}{3}}{h} = \lim_{h\to 0} \frac{\frac{4}{3}\pi\left(8+12h+6h^2+h^3\right) - \frac{32\pi}{3}}{h}$$

$$(2+h)^3 = (2+h)(4+4h+h^2)$$
$$= 8+8h+2h^2$$
$$+4h+4h^2+h^3$$
$$8+12h+6h^2+h^3$$

$$= \lim_{h\to 0} \frac{4}{3}\pi\left(12+6h+h^2\right) = \frac{4}{3}\pi(12) = \boxed{16\pi \ \frac{in^3}{in}}$$