

$$\begin{array}{r}
 10. \frac{3x^4 + x^3 - 4x^2 + 9x - 3}{x + 5} \\
 = 3x^3 - 14x^2 + 66x - 321 + \frac{1602}{x + 5} \\
 \begin{array}{r}
 -5 \overline{) 3 \quad 1 \quad -4 \quad 9 \quad -3} \\
 \underline{-15 \quad 70 \quad -330 \quad 1605} \\
 3 \quad -14 \quad 66 \quad -321 \quad 1602
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 11. \frac{5x^4 - 3x + 1}{4 - x} \\
 = -5x^3 - 20x^2 - 80x - 317 + \frac{-1269}{4 - x} \\
 \begin{array}{r}
 -4 \overline{) -5 \quad 0 \quad 0 \quad 3 \quad -1} \\
 \underline{-20 \quad -80 \quad -320 \quad -1268} \\
 -5 \quad -20 \quad -80 \quad -317 \quad -1269
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 12. \frac{x^8 - 1}{x + 2} \\
 = x^7 - 2x^6 + 4x^5 - 8x^4 + 16x^3 - 32x^2 + 64x - 128 \\
 + \frac{255}{x + 2} \\
 \begin{array}{r}
 -2 \overline{) 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1} \\
 \underline{-2 \quad 4 \quad -8 \quad 16 \quad -32 \quad 64 \quad -128 \quad 256} \\
 1 \quad -2 \quad 4 \quad -8 \quad 16 \quad -32 \quad 64 \quad -128 \quad 255
 \end{array}
 \end{array}$$

13. The remainder is $f(2) = 3$.

14. The remainder is $f(1) = -4$.

15. The remainder is $f(-3) = -43$.

16. The remainder is $f(-2) = 2$.

17. The remainder is $f(2) = 5$.

18. The remainder is $f(-1) = 23$.

19. Yes: 1 is a zero of the second polynomial.

20. Yes: 3 is a zero of the second polynomial.

21. No: When $x = 2$, the second polynomial evaluates to 10.

22. Yes: 2 is a zero of the second polynomial.

23. Yes: -2 is a zero of the second polynomial.

24. No: When $x = -1$, the second polynomial evaluates to 2.

25. From the graph it appears that $(x + 3)$ and $(x - 1)$ are factors.

$$\begin{array}{r}
 -3 \overline{) 5 \quad -7 \quad -49 \quad 51} \\
 \underline{-15 \quad 66 \quad -51} \\
 1 \overline{) 5 \quad -22 \quad -17 \quad 0} \\
 \underline{5 \quad -17} \\
 5 \quad -17 \quad 0 \\
 f(x) = (x + 3)(x - 1)(5x - 17)
 \end{array}$$

26. From the graph it appears that $(x + 2)$ and $(x - 3)$ are factors.

$$\begin{array}{r}
 -2 \overline{) 5 \quad -12 \quad -23 \quad 42} \\
 \underline{-10 \quad 44 \quad -42} \\
 3 \overline{) 5 \quad -22 \quad 21 \quad 0} \\
 \underline{15 \quad -21} \\
 5 \quad -7 \quad 0 \\
 f(x) = (x + 2)(x - 3)(5x - 7)
 \end{array}$$

$$27. 2(x + 2)(x - 1)(x - 4) = 2x^3 - 6x^2 - 12x + 16$$

$$28. 2(x + 1)(x - 3)(x + 5) = 2x^3 + 6x^2 - 26x - 30$$

$$\begin{array}{r}
 29. 2(x - 2)\left(x - \frac{1}{2}\right)\left(x - \frac{3}{2}\right) \\
 = \frac{1}{2}(x - 2)(2x - 1)(2x - 3) \\
 = 2x^3 - 8x^2 + \frac{19}{2}x - 3
 \end{array}$$

$$\begin{array}{r}
 30. 2(x + 3)(x + 1)\left(x - \frac{5}{2}\right) \\
 = x(x + 3)(x + 1)(2x - 5) \\
 = 2x^4 + 3x^3 - 14x^2 - 15x
 \end{array}$$

31. Since $f(-4) = f(3) = f(5) = 0$, it must be that $(x + 4)$, $(x - 3)$, and $(x - 5)$ are factors of f . So $f(x) = k(x + 4)(x - 3)(x - 5)$ for some constant k .

Since $f(0) = 180$, we must have $k = 3$. So $f(x) = 3(x + 4)(x - 3)(x - 5)$.

32. Since $f(-2) = f(1) = f(5) = 0$, it must be that $(x + 2)$, $(x - 1)$, and $(x - 5)$ are factors of f . So $f(x) = k(x + 2)(x - 1)(x - 5)$ for some constant k .

Since $f(-1) = 24$, we must have $k = 2$, so $f(x) = 2(x + 2)(x - 1)(x - 5)$.

33. Possible rational zeros: $\frac{\pm 1}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$; 1 is a zero.

34. Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 7, \pm 14}{\pm 1, \pm 3}$, or $\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{7}{3}, \pm \frac{14}{3}$; $\frac{7}{3}$ is a zero.

35. Possible rational zeros: $\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$, or $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$; $\frac{3}{2}$ is a zero.

36. Possible rational zeros: $\frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2, \pm 3, \pm 6}$, or $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}, -\frac{4}{3}$ and $\frac{3}{2}$ are zeros.

$$\begin{array}{r}
 37. \begin{array}{r}
 3 \overline{) 2 \quad -4 \quad 1 \quad -2} \\
 \underline{6 \quad 6 \quad 21} \\
 2 \quad 2 \quad 7 \quad 19
 \end{array}
 \end{array}$$

Since all numbers in the last line are ≥ 0 , 3 is an upper bound for the zeros of f .

$$\begin{array}{r}
 38. \begin{array}{r}
 5 \overline{) 2 \quad -5 \quad -5 \quad -1} \\
 \underline{10 \quad 25 \quad 100} \\
 2 \quad 5 \quad 20 \quad 99
 \end{array}
 \end{array}$$

Since all values in the last line are ≥ 0 , 5 is an upper bound for the zeros of $f(x)$.

$$\begin{array}{r}
 39. \begin{array}{r}
 2 \overline{) 1 \quad -1 \quad 1 \quad 1 \quad -12} \\
 \underline{2 \quad 2 \quad 6 \quad 14} \\
 1 \quad 1 \quad 3 \quad 7 \quad 2
 \end{array}
 \end{array}$$

Since all values in the last line are ≥ 0 , 2 is an upper bound for the zeros of $f(x)$.

50. Possible rational zeros: $\pm 1, \pm 3, \pm 9$. The only rational zero is -3 . Synthetic division (below) leaves $x^2 - 3$, so the irrational zeros are $\pm\sqrt{3}$.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -3 & -9 \\ & & -3 & 0 & 9 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

51. Rational: -3 ; irrational: $1 \pm \sqrt{3}$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -8 & -6 \\ & & -3 & 6 & 6 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

52. Rational: 4 ; irrational: $1 \pm \sqrt{2}$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

53. Rational: -1 and 4 ; irrational: $\pm\sqrt{2}$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -6 & 6 & 8 \\ & & -1 & 4 & 2 & -8 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

54. Rational: -1 and 2 ; irrational: $\pm\sqrt{5}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -7 & 5 & 10 \\ & & -1 & 2 & 5 & -10 \\ \hline & 1 & -2 & -5 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

63. False. $x - a$ is a factor if and only if $f(a) = 0$. So $(x + 2)$ is a factor if and only if $f(-2) = 0$.
64. True. By the Remainder Theorem, the remainder when $f(x)$ is divided by $x - 1$ is $f(1)$, which equals 3.
65. The statement $f(3) = 0$ means that $x = 3$ is a zero of $f(x)$ and that 3 is an x -intercept of the graph of $f(x)$. And it follows that $x - 3$ is a factor of $f(x)$ and thus that the remainder when $f(x)$ is divided by $x - 3$ is zero. So the answer is A.

66. By the Rational Zeros Theorem, every rational root of

$$f(x) \text{ must be among the numbers } \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}.$$

The answer is E.

67. $f(x) = (x + 2)(x^2 + x - 1) - 3$ yields a remainder of -3 when divided by either $x + 2$ or $x^2 + x - 1$, from which it follows that $x + 2$ is not a factor of $f(x)$ and that $f(x)$ is not evenly divisible by $x + 2$. The answer is B.