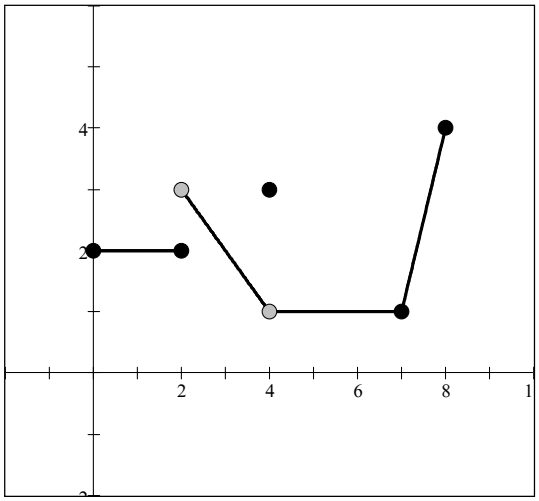
AP Calculus AB **2.1 Limits** **Name** DiMarco

Right-hand limits: $\lim_{x\to c^+} f(x)$: the limit as x approaches c from the right.

Left-hand limits: $\lim_{x\to c^-} f(x)$: the limit as x approaches c from the left.

Two-sided limits: If right and left-handed limits exist and are =, the function has a (two-sided) limit.

$\lim_{x\to c} f(x) = L \Leftrightarrow \lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L$



$f(2) =$ 2

$\lim_{x\to 2^-} f(x) =$ 2

$\lim_{x\to 2^+} f(x) =$ 3

$\lim_{x\to 2} f(x) =$ DNE

$f(4) =$ 3

$\lim_{x\to 4^-} f(x) =$ 1

$\lim_{x\to 4^+} f(x) =$ 1

$\lim_{x\to 4} f(x) =$ 1

$f(7) =$ 1

$\lim_{x\to 7^-} f(x) =$ 1

$\lim_{x\to 7^+} f(x) =$ 1

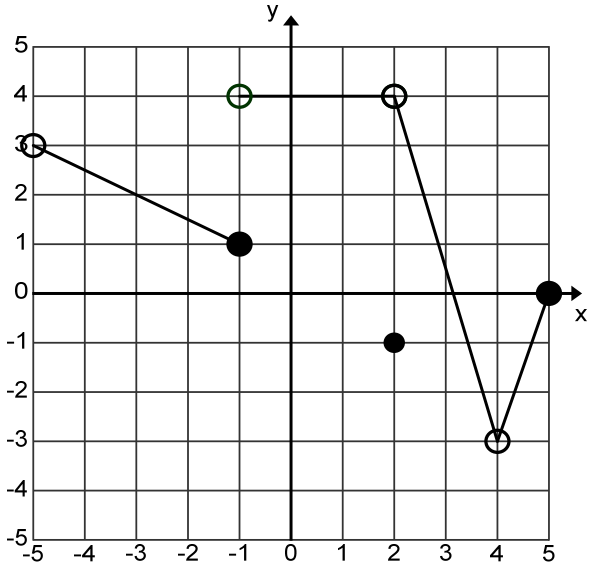
$\lim_{x\to 7} f(x) =$ 1

$f(5) =$ 1

$\lim_{x\to 5^-} f(x) =$ 1

$\lim_{x\to 5^+} f(x) =$ 1

$\lim_{x\to 5} f(x) =$ 1



$f(-1) =$ 1

$\lim_{x\to -1^-} f(x) =$ 1

$\lim_{x\to -1^+} f(x) =$ 4

$\lim_{x\to -1} f(x) =$ DNE

$f(0) =$ 4

$\lim_{x\to 0^-} f(x) =$ 4

$\lim_{x\to 0^+} f(x) =$ 4

$\lim_{x\to 0} f(x) =$ 4

$f(2) =$ -1

$\lim_{x\to 2^-} f(x) =$ 4

$\lim_{x\to 2^+} f(x) =$ 4

$\lim_{x\to 2} f(x) =$ 4

$f(4) =$ DNE

$\lim_{x\to 4^-} f(x) =$ -3

$\lim_{x\to 4^+} f(x) =$ -3

$\lim_{x\to 4} f(x) =$ -3

Use a calculator to determine the following limits:

$\lim_{x\to\infty} \frac{\sin x}{x} =$ 0

$\lim_{x\to -\infty} \frac{\sin x}{x} =$ 0

$\lim_{x\to 0} \frac{\sin x}{x} =$ 1

Polynomial Functions

Given: $f(x) = 3x^2 - 4x + 2$

Find using a calculator: $\lim_{x\to 5} f(x) =$ 57 $\lim_{x\to -2} f(x) =$ 22 $\lim_{x\to 0} f(x) =$ 2

Limits of a polynomial function may be found by evaluating the function at that value.

Notation: $\lim_{x\to c} f(x) =$ f(c)

Rational Functions

If f and g are polynomial functions, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$, provided that $g(c) \neq 0$

Properties of Limits: See handout!*

Examples: Determine the limit by substitution. No Calculators.

1. $\lim_{x\to 2} x^3 - 2x^2 + 3x - 4 = f(2)$

$= 2^3 - 2(2)^2 + 3(2) - 4$

$= 8 - 8 + 6 - 4$

$= 2$

2. $\lim_{x\to 2} \frac{x^3 - 1}{x - 1} = \frac{f(2)}{g(2)} = \frac{2^3 - 1}{2 - 1} = \frac{7}{1} = 7$

[3]

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

3. $\lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \frac{1^3 - 1}{1 - 1} = \frac{0}{0}$ ∴

Simplify: $\lim_{x\to 1} \frac{(x-1)(x^2 + x + 1)}{x - 1}$

$= \lim_{x\to 1} (x^2 + x + 1) = 1^2 + 1 + 1 = 3$

4. $\lim_{x\to 0} \frac{\tan x}{x} = \frac{\tan 0}{0} = \frac{0}{0}$ ∴

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\cos x}$

$= 1 \cdot \frac{1}{1} = 1$

5. $\lim_{x\to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4} = \lim_{x\to 0} 4 \cdot \frac{\sin 4x}{4x}$ Let $u = 4x$

$\lim_{x\to 0} 4 \cdot \lim_{u\to 0} \frac{\sin u}{u} =$

$4 \cdot 1 = 4$

6. $\lim_{x\to 0^+} \frac{|x|}{x} = 1$

7. $\lim_{x\to 0^-} \frac{|x|}{x} = -1$ So $\lim_{x\to 0} \frac{|x|}{x} =$ DNE

8. $\lim_{x\to 0} \frac{x + \sin 4x}{x} = \lim_{x\to 0} \frac{x}{x} + \lim_{x\to 0} \frac{\sin 4x}{x}$

$= \lim_{x\to 0} 1 + \lim_{x\to 0} 4 \cdot \frac{\sin 4x}{4x}$

$= 1 + 4 = 5$

A few more to try:

$$\lim_{x\to 3}\frac{3x^2-7x-6}{x^2-3x} = \frac{27-21-6}{9-9} = \frac{0}{0} \therefore$$

$$\lim_{x\to 3}\frac{(3x+2)(x-3)}{x(x-3)} =$$

$$\lim_{x\to 3}\frac{3x+2}{x} = \frac{9+2}{3} = \boxed{\frac{11}{3}}$$

$$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x} \cdot \frac{3}{3}\frac{(3+x)}{(3+x)} = \lim_{x\to 0}\frac{3-(3+x)}{3(3+x)} = \lim_{x\to 0}\frac{-x}{3(3+x)}$$

$$= \lim_{x\to 0}\frac{-x}{9+3x}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{-1}{9+3x} = \boxed{-\frac{1}{9}}$$