

1. The half-life of a substance is 4 minutes. The original mass is 100 grams. How much of the substance remains after 15 minutes?

$A(t) = A_0 \left(\frac{1}{2}\right)^{t/\text{half-life}}$
 $A = 100 \left(\frac{1}{2}\right)^{\frac{15}{4}} = 7.43 \text{ grams}$

2. Write a logistic function if:

$C = 50$

Initial value = 10, Limit to growth = 50, Passes through (2,30)

↑ not a! (0,10)

① $10 = \frac{50}{1+a \cdot b^0}$
 $10 = \frac{50}{1+a}$
 $10 + 10a = 50$

② $30 = \frac{50}{1+4b^2}$
 $30 + 120b^2 = 50$
 $120b^2 = 20$

$b^2 = \frac{1}{6}$
 $b \approx .41$

③ $f(x) = \frac{50}{1+4(.41)^x}$

Solve for x:

$a = 4$

3. $\log(x+1) - \log(2x-1) = \frac{1}{2} \log 4$

$\log \frac{x+1}{2x-1} = \log 2$

$4^{\frac{1}{2}} = \sqrt{4} = 2$

$x+1 = 4x-2$

$3 = 3x$

$x = 1$

4. $\log_4(x-3) = -1$

$4^{-1} = x-3$

$\frac{1}{4} = x-3$

$x = 3\frac{1}{4}$

5. $1 + 2e^{-2x} = 6$

$2e^{-2x} = 5$

$e^{-2x} = \frac{5}{2}$

$\ln \frac{5}{2} = -2x$

$x \approx -.458$

6. $\left(\frac{1}{4}\right)^{4x-1} = 8^{3x}$

$2^{-2(4x-1)} = 2^{3(3x)}$

$-8x + 2 = 9x$

$2 = 17x$

$\frac{2}{17} = x$

7. Write the equation of an exponential function that goes through (0,2) and (3,12). (Round b-value to nearest hundredths.)

$y = a \cdot b^x$

$12 = 2 \cdot b^3$

$b = b^3$

$b \approx 1.82$

$y = 2(1.82)^x$

$a = 2$ $x = 4$

8. The population of Chicago has been increasing at 1.2% per year. If the population is 3,000,000 in 2000, when will the population reach 3,500,000? $r = .012$

$P(t) = P_0(1+r)^t$

$3,500,000 = 3,000,000(1.012)^t$

$\frac{7}{6} = 1.012^t$

$\log_{1.012} \frac{7}{6} = t$

$t \approx 12.923 \text{ years}$