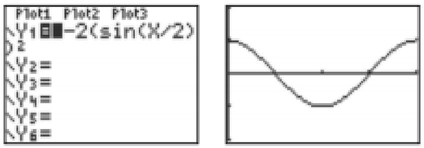
If $f = 1 - 2x^2$ and $g = \sin\left(\frac{x}{2}\right)$,

then $f \circ g = 1 - 2\left(\sin^2\left(\frac{x}{2}\right)\right) = \cos\left(2\left(\frac{x}{2}\right)\right) = \cos x$.

(The double angle formula for $\cos 2x$ is $\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$. *See Section 5.4.*) This can be seen graphically:



[0, 2π] by [−2, 2]

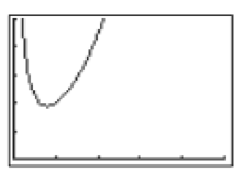
f	g	$f \circ g$
$2x - 3$	$\frac{x + 3}{2}$	x
$\lvert 2x + 4 \rvert$	$\frac{(x-2)(x+2)}{2}$	x^2
$\sqrt{x}$	x^2	$\lvert x \rvert$
x^5	$x^{0.6}$	x^3
$x - 3$	$\ln(e^3 x)$	$\ln x$
$2 \sin x \cos x$	$\frac{x}{2}$	$\sin x$
$1 - 2x^2$	$\sin\left(\frac{x}{2}\right)$	$\cos x$

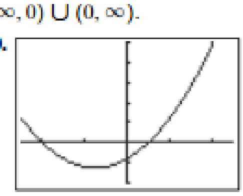
Quick Review 1.4

1. $(-\infty, -3) \cup (-3, \infty)$
2. $(1, \infty)$
3. $(-\infty, 5]$
4. $(1/2, \infty)$
5. $[1, \infty)$
6. $[-1, 1]$
7. $(-\infty, \infty)$
8. $(-\infty, 0) \cup (0, \infty)$
9. $(-1, 1)$
10. $(-\infty, \infty)$

Section 1.4 Exercises

1. $(f + g)(x) = 2x - 1 + x^2$; $(f - g)(x) = 2x - 1 - x^2$; $(fg)(x) = (2x - 1)(x^2) = 2x^3 - x^2$.
   There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.
2. $(f + g)(x) = (x - 1)^2 + 3 - x = x^2 - 2x + 1 + 3 - x = x^2 - 3x + 4$;
   $(f - g)(x) = (x - 1)^2 - 3 + x = x^2 - 2x + 1 - 3 + x = x^2 - x - 2$;
   $(fg)(x) = (x - 1)^2(3 - x) = (x^2 - 2x + 1)(3 - x)$
   $= 3x^2 - x^3 - 6x + 2x^2 + 3 - x$
   $= -x^3 + 5x^2 - 7x + 3$.
   There are no restrictions on any of the domains, so all three domains are $(-\infty, \infty)$.
3. $(f + g)(x) = \sqrt{x} + \sin x$; $(f - g)(x) = \sqrt{x} - \sin x$; $(fg)(x) = \sqrt{x}\sin x$.

   Domain in each case is $[0, \infty)$. For $\sqrt{x}$, $x \geq 0$. For $\sin x$, $-\infty < x < \infty$.
4. $(f + g)(x) = \sqrt{x + 5} + |x + 3|$;
   $(f - g)(x) = \sqrt{x + 5} - |x + 3|$;
   $(fg)(x) = \sqrt{x + 5}|x + 3|$.
   All three expressions contain $\sqrt{x + 5}$, so $x + 5 \geq 0$ and $x \geq -5$; all three domains are $[-5, \infty)$.
   For $|x + 3|$, $-\infty < x < \infty$.
5. $(f/g)(x) = \frac{\sqrt{x + 3}}{x^2}$; $x + 3 \geq 0$ and $x \neq 0$, so the domain is $[-3, 0) \cup (0, \infty)$.
   $(g/f)(x) = \frac{x^2}{\sqrt{x + 3}}$; $x + 3 \geq 0$, so the domain is $(-3, \infty)$.
6. $(f/g)(x) = \frac{\sqrt{x - 2}}{\sqrt{x + 4}} = \sqrt{\frac{x - 2}{x + 4}}$; $x - 2 \geq 0$ and $x + 4 > 0$, so $x \geq 2$ and $x > -4$; the domain is $[2, \infty)$.
   $(g/f)(x) = \frac{\sqrt{x + 4}}{\sqrt{x - 2}} = \sqrt{\frac{x + 4}{x - 2}}$; $x + 4 \geq 0$ and $x - 2 > 0$, so $x \geq -4$ and $x > 2$; the domain is $(2, \infty)$.
7. $(f/g)(x) = \frac{x^2}{\sqrt{1 - x^2}}$. The denominator cannot be zero and the term under the square root must be positive, so $1 - x^2 > 0$. Therefore, $x^2 < 1$, which means that $-1 < x < 1$. The domain is $(-1, 1)$.
   $(g/f)(x) = \frac{\sqrt{1 - x^2}}{x^2}$. The term under the square root must be nonnegative, so $1 - x^2 \geq 0$ (or $x^2 \leq 1$). The denominator cannot be zero, so $x \neq 0$. Therefore, $-1 \leq x < 0$ or $0 < x \leq 1$. The domain is $[-1, 0) \cup (0, 1]$.
8. $(f/g)(x) = \frac{x^3}{\sqrt{1 - x^3}}$. The denominator cannot be 0, so $1 - x^3 \neq 0$ and $x^3 \neq 1$. This means that $x \neq 1$. There are no restrictions on x in the numerator. The domain is $(-\infty, 1) \cup (1, \infty)$.
   $(g/f)(x) = \frac{\sqrt{1 - x^3}}{x^3}$. The denominator cannot be 0, so $x^3 \neq 0$ and $x \neq 0$. There are no restrictions on x in the numerator. The domain is $(-\infty, 0) \cup (0, \infty)$.
9. 

   [0, 5] by [0, 5]
10. 

    [−5, 5] by [−10, 25]

11. $(f \circ g)(3) = f(g(3)) = f(4) = 5$;
$(g \circ f)(-2) = g(f(-2)) = g(-7) = -6$

12. $(f \circ g)(3) = f(g(3)) = f(3) = 8$;
$(g \circ f)(-2) = g(f(-2)) = g(3) = 3$

13. $(f \circ g)(3) = f(g(3)) = f(\sqrt{3+1}) = f(2) = 2^2 + 4 = 8$;
$(g \circ f)(-2) = g(f(-2)) = g((-2)^2 + 4)$
$= g(8) = \sqrt{8+1} = 3$

14. $(f \circ g)(3) = f(g(3)) = f(9 - 3^2) = f(0) = \dfrac{0}{0+1} = 0$;
$(g \circ f)(-2) = g(f(-2)) = g\left(\dfrac{-2}{-2+1}\right)$
$= g(2) = 9 - 2^2 = 5$

15. $f(g(x)) = 3(x-1) + 2 = 3x - 3 + 2 = 3x - 1$.
Because both f and g have domain $(-\infty, \infty)$, the domain of $f(g(x))$ is $(-\infty, \infty)$.
$g(f(x)) = (3x + 2) - 1 = 3x + 1$; again, the domain is $(-\infty, \infty)$.

16. $f(g(x)) = \left(\dfrac{1}{x-1}\right)^2 - 1 = \dfrac{1}{(x-1)^2} - 1$. The domain of g is $x \neq 1$, while the domain of f is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \neq 1$, or $(-\infty, 1) \cup (1, \infty)$.
$g(f(x)) = \dfrac{1}{(x^2 - 1) - 1} = \dfrac{1}{x^2 - 2}$.
The domain of f is $(-\infty, \infty)$, while the domain of g is $(-\infty, 1) \cup (1, \infty)$, so $g(f(x))$ requires that $f(x) \neq 1$. This means $x^2 - 1 \neq 1$, or $x^2 \neq 2$, so the domain of $g(f(x))$ is $x \neq \pm\sqrt{2}$, or $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

17. $f(g(x)) = (\sqrt{x+1})^2 - 2 = x + 1 - 2 = x - 1$. The domain of g is $x \geq -1$, while the domain of f is $(-\infty, \infty)$, so the domain of $f(g(x))$ is $x \geq -1$, or $[-1, \infty)$.
$g(f(x)) = \sqrt{(x^2 - 2) + 1} = \sqrt{x^2 - 1}$. The domain of f is $(-\infty, \infty)$, while the domain of g is $[-1, \infty)$, so $g(f(x))$ requires that $f(x) \geq -1$.
This means $x^2 - 2 \geq -1$, or $x^2 \geq 1$, which means $x \leq -1$ or $x \geq 1$. Therefore the domain of $g(f(x))$ is $(-\infty, -1] \cup [1, \infty)$.

18. $f(g(x)) = \dfrac{1}{\sqrt{x} - 1}$. The domain of g is $x \geq 0$, while the domain of f is $(-\infty, 1) \cup (1, \infty)$, so $f(g(x))$ requires that $x \geq 0$ and $g(x) \neq 1$, or $x \geq 0$, and $x \neq 1$. The domain of $f(g(x))$ is $[0, 1) \cup (1, \infty)$.
$g(f(x)) = \sqrt{\dfrac{1}{x-1}} = \dfrac{1}{\sqrt{x-1}}$. The domain of f is $x \neq 1$, while the domain of g is $[0, \infty)$, so $g(f(x))$ requires that $x \neq 1$ and $f(x) \geq 0$, or $x \neq 1$ and $\dfrac{1}{x-1} \geq 0$. The latter occurs if $x - 1 \geq 0$, so the domain of $g(f(x))$ is $(1, \infty)$.

19. $f(g(x)) = f(\sqrt{1 - x^2}) = (\sqrt{1 - x^2})^2 = 1 - x^2$;
the domain is $[-1, 1]$.
$g(f(x)) = g(x^2) = \sqrt{1 - (x^2)^2} = \sqrt{1 - x^4}$;
the domain is $[-1, 1]$.

20. $f(g(x)) = f\left(\sqrt[3]{1 - x^3}\right) = \left(\sqrt[3]{1 - x^3}\right)^3 = 1 - x^3$;
the domain is $(-\infty, \infty)$.
$g(f(x)) = g(x^3) = \sqrt[3]{1 - (x^3)^3} = \sqrt[3]{1 - x^9}$;
the domain is $(-\infty, \infty)$.

21. $f(g(x)) = f\left(\dfrac{1}{3x}\right) = \dfrac{1}{2(1/3x)} = \dfrac{1}{2/3x} = \dfrac{3x}{2}$;
the domain is $(-\infty, 0) \cup (0, \infty)$.
$g(f(x)) = g\left(\dfrac{1}{2x}\right) = \dfrac{1}{3(1/2x)} = \dfrac{1}{3/2x} = \dfrac{2x}{3}$;
the domain is $(-\infty, 0) \cup (0, \infty)$.

22. $f(g(x)) = f\left(\dfrac{1}{x-1}\right) = \dfrac{1}{(1/(x-1)) + 1} =$
$\dfrac{1}{(1 + (x-1))/(x-1)} = \dfrac{1}{x/(x-1)} = \dfrac{x-1}{x}$;
the domain is all reals except 0 and 1.
$g(f(x)) = g\left(\dfrac{1}{x+1}\right) = \dfrac{1}{(1/(x+1)) - 1} =$
$\dfrac{1}{(1 + (x-1))/(x+1)} = \dfrac{1}{x/(x+1)} = \dfrac{x+1}{x}$;
the domain is all reals except -1 and 0.

23. One possibility: $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5x$.

24. One possibility: $f(x) = (x+1)^2$ and $g(x) = x^3$.

25. One possibility: $f(x) = |x|$ and $g(x) = 3x - 2$.

26. One possibility: $f(x) = 1/x$ and $g(x) = x^3 - 5x + 3$.

27. One possibility: $f(x) = x^5 + 2$ and $g(x) = x - 3$.

28. One possibility: $f(x) = e^x$ and $g(x) = \sin x$.

29. One possibility: $f(x) = \cos x$ and $g(x) = \sqrt{x}$.

30. One possibility: $f(x) = x^2 + 1$ and $g(x) = \tan x$.

31. $r = 48 + 0.03t$ in., so $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(48 + 0.03t)^3$;
when $t = 300$,
$V = \dfrac{4}{3}\pi(48 + 9)^3 = 246{,}924\pi \approx 775{,}734.6\text{ in}^3$.

32. The original diameter of each snowball is 4 in, so the original radius is 2 in. and the original volume $V = \dfrac{4}{3}\pi r^3 \approx 33.5\text{ in}^3$. The new volume is $V = 33.5 - t$, where t is the number of 40-day periods. At the end of 360 days, the new volume is $V = 33.5 - 9 = 24.5$.
Since $V = \dfrac{4}{3}\pi r^3$, we know that $r = \sqrt[3]{\dfrac{3V}{4\pi}} \approx 1.8$ in.
The diameter, then, is 2 times r, or ≈ 3.6 in.

33. The initial area is $(5)(7) = 35\text{ km}^2$. The new length and width are $l = 5 + 2t$ and $w = 7 + 2t$, so $A = lw = (5 + 2t)(7 + 2t)$. Solve $(7 + 2t)(5 + 2t) = 175$ (5 times its original size), either graphically or algebraically: the positive solution is $t \approx 3.63$ sec.

34. The initial volume is $(5)(7)(3) = 105\text{ cm}^3$. The new length, width, and height are $l = 5 + 2t$, $w = 7 + 2t$, and $h = 3 + 2t$, so the new volume is $V = (5 + 2t)(7 + 2t)(3 + 2t)$. Solve graphically $(5 + 2t)(7 + 2t)(3 + 2t) \geq 525$ (5 times the original volume): $t \approx 1.62$ sec.