10. We need
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2.$$

Section 1.2 Exercises

1. Yes, $y = \sqrt{x - 4}$ is a function of x, because when a number is substituted for x, there is at most one value produced for $\sqrt{x - 4}$.

2. No, $y = x^2 \pm 3$ is not a function of x, because when a number is substituted for x, y can be either 3 more or 3 less than x^2.

3. No, $x = 2y^2$ does not determine y as a function of x, because when a positive number is substituted for x, y can be either $\sqrt{\frac{x}{2}}$ or $-\sqrt{\frac{x}{2}}$.

4. Yes, $x = 12 - y$ determines y as a function of x, because when a number is substituted for x, there is exactly one number y which, when subtracted from 12, produces x.

5. Yes

6. No

7. No

8. Yes

9. We need $x^2 + 4 \geq 0$; this is true for all real x. Domain: $(-\infty, \infty)$.

10. We need $x - 3 \neq 0$. Domain: $(-\infty, 3) \cup (3, \infty)$.

11. We need $x + 3 \neq 0$ and $x - 1 \neq 0$. Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.

12. We need $x \neq 0$ and $x - 3 \neq 0$. Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.

13. We notice that $g(x) = \dfrac{x}{x^2 - 5x} = \dfrac{x}{x(x - 5)}$.

As a result, $x - 5 \neq 0$ and $x \neq 0$.
Domain: $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

14. We need $x - 3 \neq 0$ and $4 - x^2 \geq 0$. This means $x \neq 3$ and $x^2 \leq 4$; the latter implies that $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.

15. We need $x + 1 \neq 0$, $x^2 + 1 \neq 0$, and $4 - x \geq 0$. The first requirement means $x \neq -1$, the second is true for all x, and the last means $x \leq 4$. The domain is therefore $(-\infty, -1) \cup (-1, 4]$.

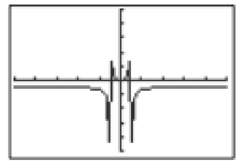
16. We need
$$x^4 - 16x^2 \geq 0$$
$$x^2(x^2 - 16) \geq 0$$
$$x^2 = 0 \quad \text{or} \quad x^2 - 16 \geq 0$$
$$x^2 \geq 16$$
$$x = 0 \quad \text{or} \quad x \geq 4, \ x \leq -4.$$

Domain: $(-\infty, -4] \cup \{0\} \cup [4, \infty)$.

17. $f(x) = 10 - x^2$ can take on any negative value. Because x^2 is nonnegative, $f(x)$ cannot be greater than 10. The range is $(-\infty, 10]$.

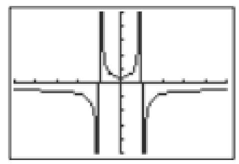
18. $g(x) = 5 + \sqrt{4 - x}$ can take on any value ≥ 5, but because $\sqrt{4 - x}$ is nonnegative, $g(x)$ cannot be less than 5. The range is $[5, \infty)$.

19. The range of a function is most simply found by graphing it. As our graph shows, the range of $f(x)$ is $(-\infty, -1) \cup [0, \infty)$.



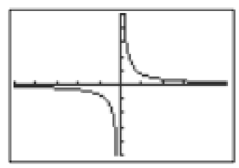
$[-10, 10]$ by $[-10, 10]$

20. As our graph illustrates, the range of $g(x)$ is $(-\infty, -1) \cup [0.75, \infty)$.



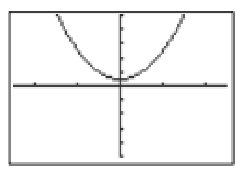
$[-10, 10]$ by $[-10, 10]$

21. Yes, nonremovable.



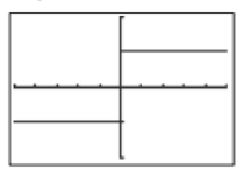
$[-10, 10]$ by $[-10, 10]$

22. Yes, removable.



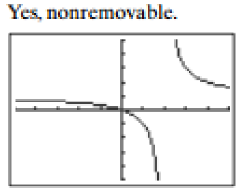
$[-5, 5]$ by $[-10, 10]$

23. Yes, nonremovable.



$[-10, 10]$ by $[-2, 2]$

24. Yes, nonremovable.



$[-5, 5]$ by $[-5, 5]$

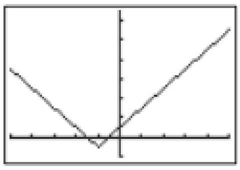
25. Local maxima at $(-1, 4)$ and $(5, 5)$, local minimum at $(2, 2)$. The function increases on $(-\infty, -1]$, decreases on $[-1, 2]$, increases on $[2, 5]$, and decreases on $[5, \infty)$.

26. Local minimum at $(1, 2)$, $(3, 3)$ is neither, and $(5, 7)$ is a local maximum. The function decreases on $(-\infty, 1]$, increases on $[1, 5]$, and decreases on $[5, \infty)$.

27. $(-1, 3)$ and $(3, 3)$ are neither. $(1, 5)$ is a local maximum, and $(5, 1)$ is a local minimum. The function increases on $(-\infty, 1]$, decreases on $[1, 5]$, and increases on $[5, \infty)$.

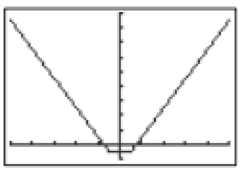
28. $(-1, 1)$ and $(3, 1)$ are local minima, while $(1, 6)$ and $(5, 4)$ are local maxima. The function decreases on $(-\infty, -1]$, increases on $[-1, 1]$, decreases on $(1, 3]$, increases on $[3, 5]$, and decreases on $[5, \infty)$.

29. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$.



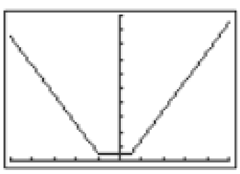
$[-10, 10]$ by $[-2, 18]$

30. Decreasing on $(-\infty, -1]$; constant on $[-1, 1]$; increasing on $[1, \infty)$.



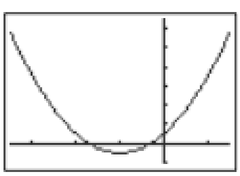
$[-10, 10]$ by $[-2, 18]$

31. Decreasing on $(-\infty, -2]$; constant on $[-2, 1]$; increasing on $[1, \infty)$.



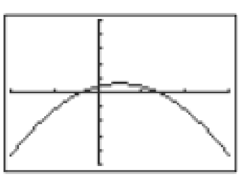
$[-10, 10]$ by $[0, 20]$

32. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$.



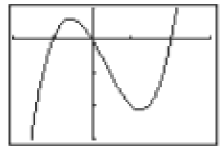
$[-7, 3]$ by $[-2, 13]$

33. Increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$.



$[-4, 6]$ by $[-25, 25]$

34. Increasing on $(-\infty, -0.5]$; decreasing on $[-0.5, 1.2]$, increasing on $[1.2, \infty)$. The middle values are approximate —they are actually at about -0.549 and 1.215. The values given are what might be observed on the decimal window.



$[-2, 3]$ by $[-3, 1]$

35. Constant functions are always bounded.

36.

$$\begin{aligned} x^2 &> 0 \\ -x^2 &< 0 \\ 2 - x^2 &< 2 \end{aligned}$$

y is bounded above by $y = 2$.

37. $2^x > 0$ for all x, so y is bounded below by $y = 0$.

38. $2^{-x} = \dfrac{1}{2^x} \geq 0$ for all x, so y is bounded below by $y = 0$.

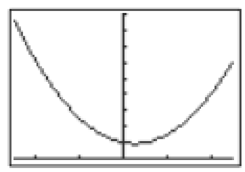
39. Since $y = \sqrt{1 - x^2}$ is always positive, we know that $y \geq 0$ for all x. We must also check for an upper bound:

$$\begin{aligned} x^2 &> 0 \\ -x^2 &< 0 \\ 1 - x^2 &< 1 \\ \sqrt{1 - x^2} &< \sqrt{1} \\ \sqrt{1 - x^2} &< 1 \end{aligned}$$

Thus, y is bounded.

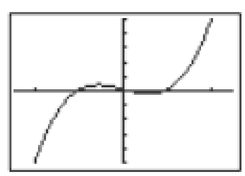
40. There are no restrictions on either x or x^3, so y is not bounded above or below.

41. f has a local minimum when $x = 0.5$, where $y = 3.75$. It has no maximum.



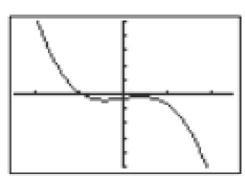
$[-5, 5]$ by $[0, 36]$

42. Local maximum: $y \approx 4.08$ at $x \approx -1.15$.
Local minimum: $y \approx -2.08$ at $x \approx 1.15$.



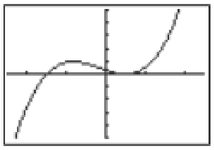
$[-5, 5]$ by $[-50, 50]$

43. Local minimum: $y \approx -4.09$ at $x \approx -0.82$.
Local maximum: $y \approx -1.91$ at $x \approx 0.82$.



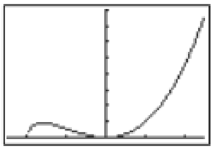
$[-5, 5]$ by $[-50, 50]$

44. Local maximum: $y \approx 9.48$ at $x \approx -1.67$.
Local minimum: $y = 0$ when $x = 1$.



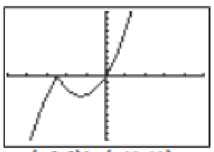
[−5, 5] by [−50, 50]

45. Local maximum: $y \approx 9.16$ at $x \approx -3.20$.
Local minima: $y = 0$ at $x = 0$ and $y = 0$ at $x = -4$.



[−5, 5] by [0, 80]

46. Local maximum: $y = 0$ at $x = -2.5$.
Local minimum: $y \approx -3.13$ at $x = -1.25$.



[−5, 5] by [−10, 10]

47. Even: $f(-x) = 2(-x)^4 = 2x^4 = f(x)$

48. Odd: $g(-x) = (-x)^3 = -x^3 = -g(x)$

49. Even: $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

50. Even: $g(-x) = \dfrac{3}{1 + (-x)^2} = \dfrac{3}{1 + x^2} = g(x)$

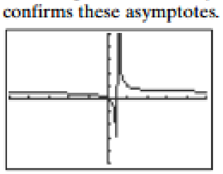
51. Neither: $f(-x) = -(-x)^2 + 0.03(-x) + 5 = -x^2 - 0.03x + 5$, which is neither $f(x)$ nor $-f(x)$.

52. Neither: $f(-x) = (-x)^3 + 0.04(-x)^2 + 3 = -x^3 + 0.04x^2 + 3$, which is neither $f(x)$ nor $-f(x)$.

53. Odd: $g(-x) = 2(-x)^3 - 3(-x)$
$= -2x^3 + 3x = -g(x)$

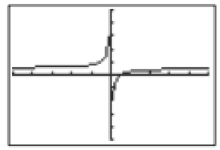
54. Odd: $h(-x) = \dfrac{1}{-x} = -\dfrac{1}{x} = -h(x)$

55. The quotient $\dfrac{x}{x-1}$ is undefined at $x = 1$, indicating that $x = 1$ is a vertical asymptote. Similarly, $\lim\limits_{x\to\infty} \dfrac{x}{x-1} = 1$, indicating a horizontal asymptote at $y = 1$. The graph confirms these asymptotes.



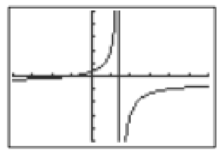
[−10, 10] by [−10, 10]

56. The quotient $\dfrac{x-1}{x}$ is undefined at $x = 0$, indicating a possible vertical asymptote at $x = 0$. Similarly, $\lim\limits_{x\to\infty} \dfrac{x-1}{x} = 1$, indicating a possible horizontal asymptote at $y = 1$. The graph confirms these asymptotes.



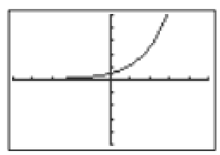
[−10, 10] by [−10, 10]

57. The quotient $\dfrac{x+2}{3-x}$ is undefined at $x = 3$, indicating a possible vertical asymptote at $x = 3$. Similarly, $\lim\limits_{x\to\infty} \dfrac{x+2}{3-x} = -1$, indicating a possible horizontal asymptote at $y = -1$. The graph confirms these asymptotes.



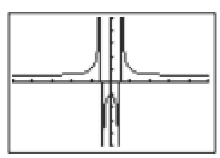
[−8, 12] by [−10, 10]

58. Since $g(x)$ is continuous over $-\infty < x < \infty$, we do not expect a vertical asymptote. However, $\lim\limits_{x\to-\infty} 1.5^x = \lim\limits_{x\to\infty} 1.5^{-x} = \lim\limits_{x\to\infty} \dfrac{1}{1.5^x} = 0$, so we expect a horizontal asymptote $y = 0$. The graph confirms this asymptote.



[−10, 10] by [−10, 10]

59. The quotient $\dfrac{x^2+2}{x^2-1}$ is undefined at $x = 1$ and $x = -1$. So we expect two vertical asymptotes. Similarly, the $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^2-1} = 1$, so we expect a horizontal asymptote at $y = 1$. The graph confirms these asymptotes.



[−10, 10] by [−10, 10]

60. We note that $x^2 + 1 \geq 0$ for $-\infty < x < \infty$, so we do not expect a vertical asymptote. However, $\lim\limits_{x\to\infty} \dfrac{4}{x^2+1} = 0$, so we expect a horizontal asymptote at

answer is C.

75. Air pressure drops with increasing height. All the other functions either steadily increase or else go both up and down. The answer is C.

76. The height of a swinging pendulum goes up and down over time as the pendulum swings back and forth. The answer is E.

77. (a)